M. Sc. Physics Examination, 2023

(2nd Year, 1st Semester)

Condensed Matter Physics - I

Paper – PG/Sc/CBS/PHY/TH/207

Time : Two hours | Full Marks : 40

(20 marks for each group)

Use separate answerscripts for each group.

Group – A

Answer ***any Two*** questions.

1. a) Let us suppose that $\{\phi_v(r)\}$ forms a complete orthonormal set of single-particle basis states and $T=\sum_{j=1}^{N} T_j$, a sum of one-particle operators, T_j. Show that T satatisfies the relation,

$$T\left|\phi_{v_{n_1}}(r_1)\cdots\phi_{v_{n_N}}(r_N)\right\rangle=\sum_j\sum_{v_b v_a} T_{v_b v_a}\delta_{v_a,v_{n_j}}$$
$$\left|\phi_{v_{n_1}}(r_1)\cdots\phi_{v_b}(r_j)\cdots\phi_{v_{n_N}}(r_N)\right\rangle$$

where $T_{v_b v_a}=\left\langle\phi_{v_b}(r_j)\middle|T_j\middle|\phi_{v_a}(r_j)\right\rangle$.

b) Show that in the second quantization representation, T could be expressed as $T=\sum_{v_a v_b} T_{v_a v_b} c_{v_a}^{\dagger} c_{v_b}$.

c) By using the single-particle momentum operator, $p=-i\hbar\nabla$, in first quantization, obtain the second quantized many-particle momentum operator in the momentum eigenket basis states, *i.e.*,

$$p = \sum_k \hbar k \; a_k^\dagger a_k \, .$$

d) Show that upon orthonormal transformation from one to another complete single particle basis states, the commutation relation remains unchanged.

e) Show that the second quantized form of density operator $\rho(r) = \delta(r - r')$ in momentum eigenket representation is

$$\rho(r) = \frac{1}{V_0}\sum_q \left(\sum_k c_k^\dagger c_{k+q}\right) e^{iq\cdot r} = \frac{1}{V_0}\sum_q \rho(q) e^{iq\cdot r}$$

where $\rho(q) = \sum_k c_k^\dagger c_{k+q}$ is the Fourier component of $\rho(r)$. 2+2+2+2+2=10

2. Consider the SSH model,

$$H_{\text{SSH}} = \sum \left[t\left(a_n^\dagger b_n + b_n^\dagger a_n\right) + t'\left(b_n^\dagger a_{n+1} + a_{n+1}^\dagger b_n\right)\right].$$

a) Express the Hamiltonian in the momentum space and obtain the expressions of (i) eigenvalues and (ii) eigenvectors.

b) Show that the expression of Pancharatnam-Berry phase of one eigenvector is

$$\gamma = \frac{1}{2}\oint \frac{t'^2 + tt'\cos k}{t^2 + t'^2 + 2tt'\cos k} dk$$

b) Plot graphically the variation of density of state functions for (i) Two dimensional solid, (ii) One dimensional solid and (iii) Zero dimensional solid; Explain the physical significance of such plots.

c) Obtain an expression for the Fermi level of a 2 dimensional solid at absolute zero considering its density of state function.

d) Describe briefly the principle of measurement of density of state of a material experimentally.

4+2+2+2

3. a) How would you geometrically obtain the reciprocal points of a direct lattice? Prove that the reciprocal points really form a lattice. What are the advantages in designing reciprocal lattice space?

b) Describe Ewald's sphere construction for X-ray diffraction in reciprocal lattice space. Obtain the condition for X-ray diffraction to occur from such a construction.

Hence show that the Bragg diffraction condition can be expressed as relation $2\vec{K}.\vec{g} + g^2 = 0$; where the symbols have their usual meanings.

c) Find the Miller indices of a plane passing through the three points (0,0,1) (1,0,0) and $\left(\frac{1}{2},\frac{1}{2},\frac{1}{4},\right)$.

4+4+2

Group – B

Answer ***any Two*** questions.

1. a) For a one-dimensional potential explain the following schemes for drawing E-K diagrams in a solid.
   - i) Periodic zone scheme
   - ii) Reduced zone scheme
   - iii) Extended zone scheme

   Show the first and second Brillouin zone for the above schemes. What is Wigner-Seitz cell?

   b) Construct the first four Brillouin Zones for a simple cubic lattice in two dimensions.

   c) The energy vs. wave vector $(E \sim k)$ relationship near the bottom of a band for a solid can be approximated as $E = A(Ka)^2 + B(Ka)^4$; where the lattice constant $a = 2{\cdot}1$ Å. The values of A and B are $6{\cdot}3\times10^{-19}$ J and $3{\cdot}2\times10^{-20}$ J respectively. Find the ratio of the effective mass of the electron to the mass of free electron at the bottom of conduction band.

   6+2+2

2. a) Define density of state function. Show that the density of state function depends upon dimension 'd' of the solid by the following relation $D(E) \propto E^{\frac{d}{2}-1}$. Is it possible 'd' to be of fractional value?

   c) By evaluating the integration in the complex plane show that $\gamma = \pi$, when $t < t'$ and $\gamma = 0$, otherwise.

   (2+2)+3+3=10

3. a) By considering the three dimensional ferromagnetic Heisenberg Hamiltonian,

$$H = -J\sum_{j,\delta} S_j \cdot S_{j+\delta}$$

   where the spins are interacting with the nearest neighbour sites (δ) only and introducing the following bosonic representation of the spin operators,

$$S^+ = \sqrt{2S}a,\ S^- = \sqrt{2S}a^\dagger,\ S^z = S - a^\dagger a,$$

   obtain the magnon dispersion relation, E(k), for a simple cubic crystal.

   b) Show that for ferromagnetic magnons, $E(k) \propto k^2$, when $k \to 0$.

   c) Show that for ferromagnetic magnons, variation of magnetization, M with temperature, T follows the Bloch $T^{\frac{3}{2}}$ law,

$$M(T) = M(0)\left(1 - \mathcal{A}T^{\frac{3}{2}}\right)$$

   where $\mathcal{A}$ is a constant.

   5+2+3=10