M. Sc. Physics Examination, 2023

(2nd Year, 2nd Semester)

Quantum Field Theory

Paper – 303

Time : 2 hours Full Marks : 40

Use separate answer script for each group.

Group – A

Answer ***any two*** questions.

1. a) Using a linear superposition of space-dependent wave-functions with time-dependent coefficients in a general time-dependent Schrödinger equation, find the algebraic form of the time-dependence of these coefficients.

   b) Show that the expectation value of the Hamiltonian evaluated with respect to a general time-dependent wave-function is independent of time. 4+6

2. Use the quantum evolution equation for an arbitrary operator to show that a bosonic as well as a fermionic operator both obey the same equation of motion. 10

3. a) Explain how the Pauli's exclusion principle follows from the anti-commutation rules obeyed by the fermionic operators.

   b) By determining the eigenvalues of the operator $\hat{N}_k \hat{f}_k$ where $\hat{N}_k$ represents the fermionic number

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operator and $\hat{f}_k$ denotes an arbitrary ferionic operator, explain the action of the operator $\hat{f}_k$ on a state given by $|n_1 n_2 ... n_k ...\rangle$. 5+5

Group – B

Answer any ***Four*** questions. 4×5=20

1. Consider the Hamiltonian $H = H_0 + H_{\text{int}}$ and define interaction picture field

$$\phi_I(t,\vec{x}) = e^{iH_0(t-t_0)}\phi(t_0,\vec{x})e^{-iH_0(t-t_0)} \text{ when } H_{\text{int}} = 0.$$

Now express the full field $\phi(t,\vec{x})$ in the form $U^\dagger(t,t_0)\phi_I(t,\vec{x})U(t,t_0)$ and find the explicit expression for $U(t,t_0)$. Then obtain a differential equation of $U(t,t_0)$ in terms of H_{int} and find the unique solution for $U(t,t_0)$ considering $U(t_0,t_0)=1$.

2. Define S matrix. State Wick's theorem. Using Wicks's theorem evaluate the expression for $\langle 0|T\{\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\}|0\rangle$ and then express the result in terms of Feynman diagrams with detailed description.

3. Write down necessary mathematical descriptions that describe the quantization in quantum mechanics and then generalize this description to quantize a theory of fields.

Interpret what these following expressions are physically expressing: i) $\phi(\vec{x})|0>$, and ii) $<0|\phi(\vec{x})|0>$.

4. Prove that $\theta(x^0 - y^0)\langle 0|[\phi(x_1),\phi(x_2)]|0\rangle$ is a Green's function of the Klein-Gordon operator.

5. State Noether's theorem. Show that the conserved charge associated with time translations is the Hamiltonian.