c) Find the fixed point(s) for the recursion relation derived in (b). Starting from a finite non zero *K*, show how the relevant fixed point is approached by successive approximations.

d) Derive the free energy per lattice site as,

$$\tilde{f}^{(n)}(K) = -\sum_{j=1}^{n} \frac{1}{2^{j-1}} g\left(K^{(j)}\right) - \frac{1}{2^{n}} \log 2$$

Mathematical steps and logical argument must be presented clearly. 2+2+3+3

Ex/SC/PHY/PG/CORE/TH/104/2023

M. Sc. Physics Examination, 2023

(1st Year, 1st Semester)

Paper – PHY/PG/CORE/TH/104

[Statistical Mechanics]

Time : Two hours Full Marks : 40

Use separate script for each group.

Group – A

Answer ***any two*** questions from Group A.

1. a) The study of density fluctuations leads to the concept of corrleation function. How does one measure the correlation function? Show that the pair correlation function is related to the compressibility of the system. 2+4

   b) Define Urshell function and explain its physical significance. Starting from Ursell function derive the expression for scattering intensity. 4

2. a) Write down Langevin equation of the particles executing Brownian motion and explain the significance of the forces that Langevin introduced in the equation. Show that for time $t >> \tau$ (Characteristics time for Brownian particle), root mean square displacement is proportional to the square root of the time. Obtain the Einstein relation between the coefficient of diffusion and the mobility. 2+4+2

[Turn over

b) Derive the diffusion equation from Fick's law. 2

3. a) State the postulates of quantum statistical mechanics. Write down the properties of density matrix. 2+2

b) Consider an electron which possesses an intrinsic spin $\frac{1}{2}\hbar\hat{\sigma}$ and a magnetic moment μ_B, where $\hat{\sigma}$ is the Pauli's spin operator, placed in a magnetic field along z direction.

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \ \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \text{ and } \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Establish the density matrix of the system and hence derive the average energy of the electron. 3

c) Derive the expression for partition function of an ideal Fermi Gas. 3

Group – B

Answer ***any two*** questions from Group B.

4. a) From symmetry argument obtain Landau free energy for a magnetic system.

b) Show that minimization of Landau free energy can demonstrate phase transition. Find expression for the order parameter as function of temperature, when Landau free energy is minimized.

c) Show that on minimization, the free energy remains continuous but its higher derivative is discontinuous across the transition temperature.

d) Hence find the jump in the heat capacity. 2+4+3+1

5. a) For spin $\frac{1}{2}$ Ising spins in a three dimensional simple cubic lattice, define "*the coordination number*", "*the short range order*" and "*the long range order*".

b) What is Bragg-Williams approximation? Discuss its implication.

c) Given that under Bragg-Williams approximation the canonical partition function is,

$$Z(h,T) = \sum_{L=-1}^{+1} \frac{N!}{\left[\frac{1}{2}N(1+L)\right]!\left[\frac{1}{2}N(1-L)\right]!} e^{\beta N\left(\frac{1}{2}\epsilon\gamma L^2 + hL\right)}$$

Show that, in absence of external field, (i.e., $h=0$), there exists a temperature T_c, such that $L = \pm L_0$ for $T < T_c$, and $L = 0$ for $T > T_c$. 3+3+4

6. Consider a one dimensional Ising chain with periodic boundary conditon.

a) Argue clearly, how Kadanoff's decimation transformation, transforms the system to an equivalent Ising system with lesser number of lattice sites.

b) Derive a recursion relation between the old and the transformed system.