Ref. SC/PHY/PG/Core/PHY/TH/105/2023

Master of Science (Physics) Day 2023

(1st Year, 2nd Semester)

Quantum Mechanics-II

Paper: Core/PHY/TH/105

Time: Two hours **Full Marks: 40**

Use separate answer script for each group.

Group A: Answer any two questions (2x10).

1. (a) Write down the Lippmann-Schwinger (LS) equation used for the description of the scattering phenomenon. (b) With the help of the LS equation and using the position basis derive the following integral equation

$$<x|\psi^{\pm}> = <x|\phi> - 2m/\hbar^2 \int d^3x' \, [e^{\pm i k|q|}/4\pi|q|] \, V(x') <x'|\psi^{\pm}>$$

with $|q|=|x-x'|$. Here symbols have usual meaning. 2+8

2. (a) Show that symmetric observables commute with the permutation operators. (b) How many transposition operators are there in a physical system consisting of three (N=3) identical particles? Following the basic action of the permutation operator explain whether any two transposition operators for N=3 commute or not. (c) Show that the decomposition of $\mathbf{P}_{312}$ is not unique. 3+4+3

3. (a) Explain why the variational method is known as an approximate method? (b) What is the basis of choosing a suitable trial function for the determination of the ground state of a harmonic oscillator using variational method? (c) Write down the trial function of the same to obtain the first excited state? (d) Derive the energy eigenvalue for the first excited state of a one dimensional harmonic oscillator by using the variational method. 2+2+1+5

[Turn over

Group B

Answer any TWO

(The symbols used below will carry their usual meaning, unless otherwise stated)

1.a) Discuss how the eigenvalues corresponding to the simultaneous eigenkets of J^2 and J_z can be determined, starting from the commutation relations $[J_i, J_k] = i\hbar\epsilon_{ikl}J_l$ and $[J^2, J_i] = 0$ $(i = x, y, z)$. Hence discuss the construction of the four Bell states corresponding to a composite system of two spin-1/2 particles.

1.b) Consider the quantum mechanical problem of a two electron atom. Discuss the basic idea of approximately solving this using the Hartree-Fock method. **[(5+2)+3]**

2.a) Consider a mixed ensemble of electrons in which 40% population is at a state $|1\rangle = \cos\theta|z+\rangle + \sin\theta e^{i\phi}|z-\rangle$, 50% is at a state $|2\rangle = \cos\theta|z+\rangle - \sin\theta e^{i\phi}|z-\rangle$ and 10% is at a state $|x+\rangle$. Compute the expectation value of S_x in this ensemble.

2.b) Check whether the vector $j_\mu = ie\left(\Phi^\dagger\partial_\mu\Phi - (\partial_\mu\Phi^\dagger)\Phi\right)$ is conserved in the Klein-Gordon theory. Here e is a real constant and Φ is the complex Klein-Gordon wave function.

2.c) Write down the Dirac equation and find its solutions in the rest and in the moving frames. Discuss the solutions representing the particle and antiparticle states. **[3+2+5]**

3.a) Show that the special relativistic covariance of the Dirac equation implies,

$$S^{-1}(\Lambda)\gamma^\mu\Lambda_\mu{}^\nu S(\Lambda) = \gamma^\nu$$

where Λ denotes the Lorentz transformations and $S(\Lambda)$ is the transformation matrix for the Dirac wave function $(\Psi \to S(\Lambda)\Psi)$. For an infinitesimal Lorentz transormation, $\Lambda^\mu{}_\nu = \delta^\mu_\nu + \omega^\mu{}_\nu$, show that the quantity $\bar{\Psi}\Psi$ transforms like a scalar. You may use $S(\Lambda) = \mathbf{I} - \frac{i}{4}\omega_{\mu\nu}\sigma^{\mu\nu}$ where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$.

3.b) Consider a bipartite state,

$$|\psi\rangle = \cos\theta|++\rangle + \sin\theta|--\rangle$$

where the first entry in each of the above two kets corresponds to Alice and the second to Bob. Find out the partially traced density matrix from Bob's point of view. Is it pure? Compute the corresponding von Neumann entropy. For what value of θ the entropy is maximum?

3.c) Consider a spin-1 operator written in the matrix representation in the diagonal basis. What will be its trace? Will it remain the same for any other matrix representation? **[(2+3)+3+2]**