spaces. Express the state vectors of one space in terms of the state vectors of other space.

5. Write down the effective potential for H-atom. Then write down the radial part of the Schrödinger equation. Find the behaviour of its solutions at the origin.

6. For any two conjugate observables $\hat{P}$ and $\hat{Q}$ show that $\Delta P \Delta Q \geq \frac{h}{2}$.

M. Sc. Physics Examination, 2023

(1st Year, 1st Semester)

Paper – PHY/PG/CORE/TH/103

[Quantum Mechanics I]

Time : Two hours Full Marks : 40

Group – A

Answer ***any two*** questions. 2×10=20

1. a) What are physical significance of $\langle X | \psi(t) \rangle$ and $\langle P | \psi(t) \rangle$? How are they related to each other?

   b) In most cases, it is easy to use the position Eigen vectors $(|X\rangle)$ as the basis vectors. Justify the statement. Give an example where both the position and momentum basis are equally easy or equally hard to use. Justify your answer.

   c) i) How do the orthonormality and the completeness condition change if one goes from the discrete basis vectors to the continuous basis vectors like position Eigen ket presentation?

   ii) Expectation value of position operator $\hat{X}$ in any state vector is given by $\langle \psi(t) | X | \psi(t) \rangle$. Show how it looks like in the corresponding position basis wave function.

d) Write down the translational operator (T_λ). Explain why the name is so. 2.5+2.5+2.5+2.5

2. a) Show that infinite superposition of the Eigen ket of number operator $\widehat{N}$ forms a coherent state $|\alpha\rangle$.

b) Find out the action of the Displacement operator $D(\alpha)$ on $|0\rangle$, the ground state of LHO.

c) Show that the Displacement operator D(α) is an Unitary operator.

d) Calculate the coherent state $|\alpha\rangle$ in position Eigen function basis. 3+2+1+4

3. a) What do you mean by squeezed state? Show that $|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$ corresponds to a squeezed state in *position* quadrature.

b) Calculate the 2nd order correction to the energy Eigen value due to time independent perturbation to the Hamiltonian. (Consider that the unperturbed states are nondegenerate) Discuss whether it is valid for degenerate unperturbed states. 3.5+6.5

Group – B

Answer question no. **1** and ***any four*** questions from the rest. 4+4×4

1. a) Write down the Schrödinger equation in terms of state vector and then in $\{\vec{r}\}$ representation.

b) Show that the eigenvalues of the unitary operator are complex numbers with modulus 1.

c) Show that $\hat{S}_Z$ of a spin 1/2 system forms a C.S.C.O. Find the matrix reprsentation of $\hat{S}_Z$.

d) Write down the boundary conditions to solve the Schrödinger equation for H-atom.

e) What are the possible results of the measurements of a physical quantity of a quantum system?

f) Describe the state of the system immediately after the measurement.

g) State Ehrenfest theorem.

h) State Schwartz inequality.

2. Define state vectors in Schrödinger picture, Heisenberg picture and in interaction picture. From the definition of the state vector in the interaction picture derive its time evolution equation. Then solve this equation to find the state vector.

3. Describe how one can construct an observable in quantum mechanics from a classically defined physical quantity. Show that the Parity operator is hermitian and unitary.

4. Construct all possible sets of C.S.C.O. for a system of two spin 1/2 particles. Describe the corresponding state