$$Q_{ij} = \int d\vec{r}'\rho(\vec{r}')\left[x_i'x_j' - \frac{1}{3}r'^2\delta_{ij}\right]$$

is independent of the origin (the primed coordinates, as usual, denote the source coordinates). 12+8=20

2. a) Consider two concentric conducting spheres of radii R_1 and R_2, with $R_1 < R_2$. A charge Q is given on the inner sphere while the outer sphere is maintained at a potential Φ_0. The space between these spheres is filled with a dielectric medium of permittivity $\in$. Find the bound charge densities at the inner and outer surfaces of the dielectric medium.

   b) A circular ring of radius R is placed with its centre at the origin. A charge of density $\lambda = \lambda_0 \sin\phi$ is given on the ring, where the angle ϕ is being measured from the X-axis and λ_0 is a positive constant. Find the dipole moment of this charged system with respect to the point having coordinates $(0, -R/2)$.

14+6=20

Ex/SC/PHY/PG/CORE/TH/106/2023

M. Sc. Physics Examination, 2023

(1st Year, 2nd Semester)

Paper – PHY/PG/CORE/TH/106

[Electrodynamics]

Time : Two hours | Full Marks : 40

Use separate answer script for each group.

Group – A

Answer ***any two*** questions.

1. a) Considering a second-rank antisymmetric tensor $t^{\mu\nu}$ write down its transformation rule from S' to S frame. Hence show that t^{03} component transforms satisfying the following relation $t^{03'} = \gamma\left(t^{03} + \beta t^{31}\right)$.

   b) Show that the Lorentz force law in relativistic notation can be expressed as $K^\mu = q\eta_\nu F^{\mu\nu}$; where the symbols have their usual meanings.

   c) Define proper acceleration α_μ. Obtain its transformation relations from S' to S frame. Show that $\eta^\mu\alpha_\mu = 0$.

   d) Show that the x component of ordinary force vectors transformations according to the following equations

$$F_x' = \frac{F_x - \beta(\boldsymbol{U}.\boldsymbol{F})/c}{\left(1 - \beta\dfrac{u_x}{c}\right)}$$

e) Find the matrix describing a Lorentz transformation with velocity v along x axis followed by a Lorentz transformation with velocity $\overline{v}$ along the y axis. Does it matter in what order the transformations are carried out? 5×2

2. a) The electric and magnetic fields due to an oscillating electric dipole are given by

$$\vec{E} = -\frac{\mu_0 p_0 \omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos\omega\left(t-\frac{r}{c}\right)\hat{\theta}$$

$$\vec{B} = -\frac{\mu_0 p_0 \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos\omega\left(t-\frac{r}{c}\right)\hat{\phi}$$

Calculate the Poynting vector for the above dipole and also obtain an expression for the total power radiated by the above dipole.

Plot the intensity profile of the dipole graphically.

b) Suppose a point charge q is moving in a specified trajectory and you have to calculate the scalar and vector potential at a point P. Can two points on the trajectory communicate with P at a particular time t? Justify your answer.

Show that the scalar potential $V(\vec{r},t)$ and vector potential $\vec{A}(\vec{r},t)$ can be expressed as

$$\vec{A}(\vec{r},t) = \frac{\vec{v}}{c^2}V(\vec{r},t)$$

(where the symbols have their usual meanings) 4+6

3. a) What are electromagnetic field tensor $F^{\mu\nu}$ and dual tensor $G^{\mu\nu}$? How many components $F^{\mu\nu}$ have? How many are independent components?

Show that Maxwell's equations can be expressed as following:

$$\sum_0^3 \frac{\partial F^{\mu\nu}}{\partial x^\nu} = \mu_o J^\mu \text{ and } \sum_0^3 \frac{\partial G^{\mu\nu}}{\partial x^\nu} = 0$$

(where the symbols have their usual meanings)

b) What is current density 4-vector? Show that continuity equation can be expressed as

$$\sum_0^3 \frac{\partial J^\mu}{\partial x^\mu} = 0$$

(where the symbols have their usual meaning)

8+2

Group – B

(20 marks)

Answer ***any one*** question.

1. a) A point charge q is placed at a distance d from the centre of a grounded conducting sphere of radius R, where $R < d$. Find the total charge induced on the surface of the sphere.

b) Find the condition(s) under which the quadrupole moment tensor given by