6. a) Deduce Euler – Lagrange's equations of motion from Hamilton's variational principle. Hence apply the variational principle to show that the shortest distance between two points in a plane is a straight line.

b) Show that the generating function $F = \sum q_k P_k$ generates identity transformation. Comment on the significance of the result you obtain. (Here the symbols q_k and P_k carry usual meanings.

4+3+(2+1)=10

Ex/SC/PHY/PG/CORE/TH/101/2023

M. Sc. Physics Examination, 2023

(1st Year, 1st Semester)

Paper – PHY/PG/CORE/TH/101

[Classical Mechanics]

Time : Two hours Full Marks : 40

Use separate script for each Group.

Group – A

Answer ***any two*** questions from Group-A.

1. a) The distance between any two points on a rigid body remains constant. Identify the correct degree of freedom for general motion of such a body.

b) For a rigid body moving with one point fixed, define Euler angles. Sketch necessary diagrams, clearly showing the angles and axes.

c) Obtain a general rotation matrix $R(\phi, \theta, \psi)$, be achieved by a single rotation about a "suitable axis"? Justify your answer. 2+3+3+2

2. Consider an inertial laboratory frame and a rotating frame with common origin. Vector $\vec{r}$ is represented in the two frames as $\vec{r} = \sum_{i=1}^{3} x_i \cdot \hat{e}_i = \sum_{i=1}^{3} x_i' \cdot \hat{e}_i'$. x_i and x_i' are the components of vector $\vec{r}$, and $\hat{e}_i$ and $\hat{e}_i'$ are the instantaneous unit vectors along the axes in the laboratory frame and the rotating frame respectively.

[Turn over

a) Find the equation connecting velocities in the two frames.

b) Find the equation connecting accelerations in the two frames.

c) Show that Newton's equation of motion in the rotating frame, requires introduction of fictitious forces. Discuss physical situations where they may appear. 3+3+4

3. a) Obtain the phase portrait of a two dimensional system $\dot{x} = x^2 - x$, $\dot{y} = -y$ in a region including the fixed points.

b) Analyse the following two dimensional equations and identify the bifurcation(s) that occur(s) with change in the parameter μ. $\dot{r} = \mu \cdot r - 4r^3 + 4r^5$ and $\dot{\theta} = \omega$. 4+6

Group – B

Answer ***any two*** questions from Group-B.

4. a) What do you mean by the term "degress of freedom" and "generalized coordinates"? How many degrees of freedom a rigid body has when it rotates about a fixed axis in space?

b) Consider the motion of a particle of mass "m" moving in space. Using the cylindrical coordinates (ρ, φ, z) as the generalized coordinates, estimate the generalized force component if a force **F** acts on it.

c) For a scleronomic system, show that the kinetic energy is a homogeneous quadratic of generalized velocities.

d) A particle of unit mass moves in the xy plane in such a way that it obeys the following equations:

$$\dot{x}(t) = y(t) \text{ and } \dot{y}(t) = -x(t)$$

If the particle is considered to be moving in a conservative force field, frame the Lagrangian of the system. (2+1)+3+2+2=10

5. a) Obtain the equation of the curve $y = ax^3$ (where a is a constant) in the Legendre transformormed form. Reconstruct the standard equation of the curve $y = ax^3$ from its Legendre transform form. Hence show that the Hamiltonian of a system in phase space is connected with its Lagrangian in the configuration space through Legendre transformation.

b) If F(q, p, t) and G(q, p, t) are two integrals of motion, then show [F, G] is also an integral of motion. [*Jacobi's identity may be used without any formal proof*] (Here the symbols q, p and t carry usual meanings). 2+2+2+4=10