B Sc Physics 3rd Year 1st Sem Supplementary Examination 2023

Paper: Condensed Matter Physics

(Subject code: UG/SC/DSE/PHY/TH/01/A2)

Time: Two hours Full marks: 40

(20 marks for each group)

Use separate answer-scripts for each group.

Group A

Answer any FOUR questions.

$4 \times 5 = 20$

1. What do you mean by lattice? How is it different from crystal? What is primitive and non-primitive unit cell? Explain with proper diagram. [1+1+3=5]

2. Name seven crystal systems? Differentiate them with their lattice parameters 'a', 'b', 'c' and 'α', 'β', 'γ'. Which crystal system is most symmetric? [1+3+1=5]

3. Explain with diagram n-fold rotational symmetry. Which value of n is not permissible in symmetry of lattice system? Explain why? [2+1+2=5]

4. What do you mean by space group? Draw and explain Glide symmetry. How many possible space groups are possible in three-dimensional lattice structure? [1+3+1=5]

5. What do you mean by reciprocal lattice? Show that reciprocal lattice of simple cubic structure is also simple cubic. [2+3=5]

6. Draw lattice plane defined by Miller indices (311). Write and prove Bragg's law. [2+3=5]

7. What are nanomaterials? Show with example surface to volume ratio of nanomaterials is much more compared to bulk materials. What is two-dimensional material? [1+3+1=5]

[Turn Over

Group B

Answer any TWO questions.

$2 \times 10 = 20$

1. (a) Write down the expression of Lennard-Jones potential. Draw the variation of the potential with the separation between the atoms.

(b) Find the minimum value of that potential and location of the minimum.

(c) The total potential energy of the whole crystal can be expressed as

$$V = \frac{1}{2} \sum_{\mathrm{R,R'}}' \Phi(\mathrm{R} - \mathrm{R'} + \mathrm{u_R} - \mathrm{u_{R'}}),$$

where the symbols have their usual meaning. Now employing the three-dimensional Taylor series expansion show that

$$V \approx \frac{1}{2} \sum_{\mathrm{R,R'}}' \Phi(\mathrm{R} - \mathrm{R'}) + \frac{1}{4} \sum_{\mathrm{R,R'}}' [(\mathrm{u_R} - \mathrm{u_{R'}}) \cdot \nabla]^2 \, \Phi(\mathrm{R} - \mathrm{R'}).$$

(d) Show that harmonic part of the potential can be expressed as

$$V_{\mathrm{Harm}} = \frac{1}{2} \sum_{\substack{\mathrm{R,R'} \\ \alpha,\beta}}' \mathrm{u}_{\mathrm{R}\alpha} \, \mathcal{D}_{\alpha\beta}(\mathrm{R} - \mathrm{R'}) \, \mathrm{u}_{\mathrm{R'}\beta},$$

where $(\alpha, \beta) = x, y, z$, and,

$$\mathcal{D}_{\alpha\beta}(\mathrm{R} - \mathrm{R'}) = \delta_{\mathrm{R,R'}} \sum_{\mathrm{R''}}' \Phi_{\alpha\beta}(\mathrm{R} - \mathrm{R''}) - \Phi_{\alpha\beta}(\mathrm{R} - \mathrm{R'}).$$

[(1+1)+(1+1)+4+2=10]

2. Consider a one-dimensional Bravais lattice with diatomic basis in which the masses of two ions are m and M, $(M > m)$, and where the nearest neighbours interact through the spring constant $\mathcal{K}$.
(a) Show that the dispersion relations are

$$\omega_{\pm}^2 = \frac{\mathcal{K}}{mM}\left(m + M \pm \sqrt{m^2 + M^2 + 2mM \cos ka}\right).$$

(b) Find the ratios of amplitudes of two ions within a primitive cell in acoustic and optical branches when (A) k$a = 0$, and (B) k$a = \pi$.

(c) Draw a sketch of the dispersion relation within the first Brillouin zone.

(d) Describe the form of the dispersion relation when $M \gg m$.

[4+(1.5+1.5)+1+2=10]

3. (a) Considering the spin-1/2 degrees of freedom, the single-electron Hamiltonian can be written as

$H = -\dfrac{\hbar^2}{2\,m}\,(\sigma \cdot \nabla)^2$, where σ_α, $(\alpha = x,\ y,\ z)$ are the Pauli matrices.

Show that in the presence of a magnetic field B $(= \nabla \times \mathrm{A})$, this Hamiltonian can be expressed as

$$H = \frac{1}{2\,m}\,(-i\,\hbar\,\nabla + e\,\mathrm{A})^2 - \mu \cdot \mathrm{B}, \quad \text{where } \mu = -\frac{e\hbar\sigma}{2m}.$$

Symbols have their usual meaning.

(b) State Hund's three rules for the determination of ground state of an atom.

(c) What will be the ground state of an atom when its f-shell contains 10 electrons.

(d) Find the expression of eigenvalue E_{J} of the Hamiltonian,

$$H_{\mathrm{SO}} = \lambda \mathrm{L} \cdot \mathrm{S},$$

where

$$H_{\mathrm{SO}}|\mathrm{J}\rangle = E_{\mathrm{J}}|\mathrm{J}\rangle.$$

Symbols have their usual meaning. [3+3+2+2=10]