6. Define diffused radiation pressure? Show that diffused radiation pressure is one-third of energy density. From the idea of Fermi-Dirac statistical distribution derive the expression of highest energy possessed by fermion at absolute zero temperature. 2+3+5

Ex/SC/PHY/UG/CORE/TH/14/2023

B. Sc. Physics (Hons.) Examination, 2023

(3rd Year, 2nd Semester)

Statistical Mechanics

Paper – CORE 14

Time : 2 hours Full Marks : 40

The figures in the margin indicate full marks.

Candidates are instructed to give their answers in their own words as far as practicable.

Answer ***any four*** questions. 4×10=40

1. i) State and explain the 'central limit theorem'. 2

   ii) The probability density function (pdf) of the Gaussian distribution reads as

   $f(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}$, where μ and σ are two important parameters.

   Prove that $\langle x\rangle=\mu$. What is the significance of the other parameter σ? Under what conditions the Binomial distribution tends toward Gaussian distribution? 2+1+2

   iii) In 1-D random walk problem, a drunk man wants to return his home from the bar. His steps are random. This means his steps are equally probable to the left or to the right. But there is a finite chance of return to home. Let us focus on a quantity which is net displacement to right $m=(n_1-n_2)l$, where n_1 and n_2 are steps to the right and steps to the left, respectively, with total steps $N=n_1+n_2$. And, l is the each step length. Prove that the value of $\mathrm{var}(m)=Nl^2$. Given $\langle n_1\rangle = N/2$ and $\langle n_1^2\rangle = (N^2+N)/4$. 3

2. i) Consider a hypothetical system containing total number of particles $N=3$. Furthermore, for simplicity, consider two energy levels: one E_1 having degeneracy $g_1=2$ contains $n_1=2$ particles and other E_2 with degeneracy $g_2=2$ contains $n_2=1$ particle. Obviously, $N=n_1+n_2$.

   Enumerate the number of distinct ways if (a) the particles are distinguishable and there is no restriction on how many number of particles can be put in each quantum state, and (b) the particles are indistinguishable but you can put maximum one particle per quantum state. 2+2

   ii) Briefly mention the significance of Liouville's theorem in statistical mechanics. 2

   iii) Draw the phase space trajectory of a particle of mass m thrown vertically upwards with an initial velocity u_0. 2

   iv) Choose the correct option: The dimension of phase space of a system consists of ten rigid diatomic molecules is (a) 50 (b) 60 (c) 100 (d) 120. 2

3. i) Calculate the number of microstates of an ideal monatomic gas containing N number of molecules kept in a container of volume V and having total energy E. Hence calculate the entropy of a monatomic ideal gas to arrive at the famous 'Sackur-Tetrode' equation. 5+2

   ii) Write a short note on 'Gibbs paradox'. 3

4. Starting from Planck's idea of black body radiation show that the energy density of Planck's oscillators in frequency range v and $v+dv$ is

$$u_v dv = \frac{8\pi h v^3}{c^3\left(e^{\frac{hv}{KT}}-1\right)}dv\ .$$

   Using this energy density expression, establish Stefan-Boltzmann law of black body radiation. 6+4

5. Derive the statistical expression of most probable distribution for indistinguishable particles having integral spin. A system has 7 particles arranged in two compartments. The first compartment has 8 cells and the second has 10 cells. Calculate the number of microstates in macrostate (3, 4) when particles obey Fermi-Dirac statistics. 7+3