Ref. No. SC/PHY/UG/CORE/TH/11/2023(S)

Bachelor of Science (Physics) Special Supplementary Examinations 2023

(3rd Year, 1st Semester)

Quantum Mechanics and Applications

Time: Two hours Full Marks: 40

Group – A (20 marks)

Answer any two questions (2x10)

1. (i) What do you mean by the quantum mechanical tunnelling of a particle? (ii) How can you calculate the probability of tunnelling of a particle using the time-independent Schrodinger equation? Schematic diagram and derivation are required. (3 + 7)

2. (a) Starting from the $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ derive a generalized equation for $[\mathbf{L}_i, \mathbf{L}_j]$. (b) Explain the quantization of the z-component of the orbital angular momentum for $l = 3$. (c) Derive an expression for operator $\mathbf{L}_z$ in spherical polar coordinate system. (4+2+4)

3. (a) How do you define degenerate states? (b) Consider that a particle of mass m subjected to $V = 0$ is confined within a three dimensional box. Obtain an expression for the energy eigen value of the particle. Discuss how energy eigen value depends on the dimensions of the box. Derivation is required. (3+7)

4. (a) Write down the Hamiltonian of a hydrogen atom. (b) How do you relate, $\mathbf{p}^2$ and the Laplacian operator where $\mathbf{p}$ is the momentum operator? (c) Write down the ground state eigen function, ψ_{100} of the hydrogen atom. (d) How many quantum numbers are required to express the radial part, R, of the wave function of the hydrogen atom? (2+4+2+2)

[Turn Over

Group B

(20 Marks)

Answer any two Questions.

1. (i) Show that the lowest order relativistic correction to Hydrogen Hamiltonian is $H_r^{'} = -\frac{\hat{p}^4}{8m^3c^2}$, where $\hat{p}$ is the operator corresponding to the relativistic momentum. (ii) Further, show that the first order relativistic correction to the energy levels is $E_r^1 = -\frac{E_n^2}{2mc^2}\left[\frac{4n}{l+1/2} - 3\right]$. [5+5]

2. (i) Show that, the contribution to the energy levels due to spin-orbit interaction is $E_{so}^1 = \frac{E_n^2}{mc^2}\left[\frac{n\{j(j+1)-l(l+1)-3/4\}}{l(l+1/2)(l+1)}\right]$. (ii) Show that the fine structure breaks the degeneracy of energy levels in ℓ in Hydrogen atom, where ℓ is the quantum number corresponding to the orbital angular momemntum.

[5+5]

3. (i) Discuss the importance of Landé g-factor. (ii) Calculate Landé g-factor for the following states: $2^2D_{3/2}$, $2^2D_{5/2}$ (iii) Show the splitting of energy levels of Hydrogen atoms in a weak magnetic field is not uniformormly separated in energy. [2+3+5]