2. a) Describe the principle of estimation of the value of π using Monte Carlo simulation. Write a basic structure of a pseudocode that calculates the value of π.

   b) Explain Inverse Transform sampling method. Write an algorithmic setup to generate Random numbers according to Exponential distribution $f(x) \propto \exp(-x/3)$. 5+5=10

3. a) With the definition of Ising Model in 2d square lattice, discuss its basic properties explaining the Ferromagnetic to Paramagnetic transition.

   b) Sketch out the basic structure (psedu-code/steps) of Metropolis Algorithm to simulate Ising model on square lattice. 4+6=10

B. Sc. Physics (Hons.) Examination, 2023

(3rd Year, 2nd Semester)

Physics with Computer Simulations

Paper – DSE 4A1

Time : 2 hours Full Marks : 40

(20 marks for each group)

Prepare separate answer scripts for each group.

Group – A

Answer ***any two*** questions. 2×10=20

1. a) i) Write down the set of differential equations for the SIR model in terms of the variables $S(t)$, $I(t)$, $R(t)$, respectively, known as susceptible, infected and recovered populations at time t.

   ii) State the significance of the parameters used here.

   b) Show that the total population, $N = S(t) + I(t) + R(t)$ is a constant during the time evolution.

   c) Find the integral form of $S(t)$ and $I(t)$.

   d) i) Write down the set of differential equations for the SEIR model.

   ii) What do you mean by the term "exposed population"?

   iii) Explain the significance of additional parameter used in this model.

   (2+1)+1+3+(1+1+1)=10

2. a) Consider the motion of a simple pendulum in the vertical plane and write down the Lagrangian of the system.

   b) Using the Euler-Lagrange equation find the equation of motion of the pendulum.

   c) i) Draw a schematic phase space diagram indicating the position of fixed points, like center and hyperbolic.

   ii) Draw trajectories close to the center, and

   iii) the separatrix in the diagram.

   iv) Write down the coordinates of center and hyperbolic fixed points.

   d) i) Write down the expression of total energy in terms of phase space coordinates.

   ii) Obtain the equation of elliptic trajectory close to the center. 2+1+(1+1+1+1)+(1+2)=10

3. a) Consider the circular motion of two bodies of different masses under the mutual attractive gravitational force about their center of mass. Now introducing the dimensionless parameter μ, obtain the expression of their masses and their distances from the center of mass in terms of μ.

   b) Find the limiting values of μ.

   c) Introducing a suitable value for the period of a complete revolution, *T*, show that the value of gravitational constant G is unity in the normalized coordinate system.

   d) Consider the equation of motion for the damped simple pendulum,

$$\ddot{\theta} + 2\beta\dot{\theta} + \omega_0^2 \sin\theta = 0$$

   i) Now find the coordinates of the fixed points.

   ii) Obtaining the eigenvalues of the Jacobian matrix at the fixed points find the nature of those fixed points when the damping is very high. 2+1+2+(2+3)=10

Group – B

Answer ***any two*** questions. 2×10=20

1. a) Define Monte Carlo estimator for integration of the function $f(x)$ between range [a,b]. Show that the expectation value of the estimator converges to the actual integration value. Comment on it's variance.

   b) Consider a function $f(x) = e^{-|x-2|}$. Write a pseudo-code to estimate the integration from $x = -6$ to $x = 6$. Explain the concept of Importance Sampling in this context. 4+6=10