B. Sc. Physics (Hons.) Examination, 2023

(3rd Year, 2nd Semester)

Advance Dynamics

Paper – DSE/03/A1

Time : 2 hours Full Marks : 40

Use separate answer scripts for each group.

Group – A

Answer ***any four*** questions from Group A

(each carry 5 marks)

1. A two dimensional linear system is described by

   $\frac{dx}{dt} = \mathbf{A}\mathbf{x}$ where $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$.

   Analyze the equation to show how the nature of the fixed point depends on the determinant Δ and trace τ of the matrix **A**.

2. Find the fixed points of a nonlinear system described by,

   $\frac{dx}{dt} = -x + x^3$ and $\frac{dy}{dt} = -2y$.

   Use linearization to classify the fixed points. Show the flow lines on the *xy* plane.

3. Obtain the bifurcation diagram and identify its type, for the one-dimensional system $\frac{dx}{dt} = rx + 4x^3$.

4. Analyze and identify the bifurcation for the two dimensional system,

$$\frac{dx}{dt} = \mu - x^2 \text{ and } \frac{dy}{dt} = -y .$$

5. What is a limit cycle? Sketch the flow lines for the system,

$$\frac{dr}{dt} = r\left(1-r^2\right)\left(9-r^2\right) \text{ and } \frac{d\theta}{dt} = 2 .$$

Group – B

Answer ***any two*** questions.

1. a) Explain the concept of continuum in the study of fluid mechanics.

   b) Explain material volume and control volume of fluid in connection with the description of fluid flows.

   c) The velocity field in two dimensional flow is given by $\vec{V} = 2xy^2\hat{i} + 3xyt\hat{j}$ in m/s. Find the magnitude of acceleration of a fluid particle located at $x = 1\,\text{m}$, $y = 1\,\text{m}$ at time $t = 1\,\text{s}$.

   d) An idealized incompressible flow has the proposed three dimensional velocity distribution $\vec{V} = 4xy\hat{i} + f(y)\hat{j} - zy^2\hat{k}$. Find the appropriate form of the function $f(y)$ that satisfies the continuity equation.

   e) What is the significance of Reynold's number?

2+2+2+2+2

2. a) Write down Navier-Stokes equations in three dimensions in Cartesian co-ordinates system, explaining all the terms involved in the equations.

   b) Use Navier-Stokes equations to derive fully-developed plane Poiseuille flow between two stationary plates having constant pressure gradient. Draw the flow profile and hence find the maximum velocity of the flow. Find also the shear stress for this flow and explain its variation across the flow.

2+(4+2+2)

3. Rabbits are introduced into an area of natural grassland where they come into competition with sheep. A model is proposed for this process as follows:

$$\dot{x} = x(3 - x - 2y)$$

$$\dot{y} = y(2 - x - y)$$

where x and y are the populations of rabbits and sheep respectively.

   a) Explain the assumptions of the given model. Why is it suitable for this situations?

   b) Find the fixed points and investigate their stability.

   c) Draw the phase portrait and comment on the biological implications.

2+5+3