Ex/SC/PHY/UG/CORE/TH/08/2023

B. Sc. Physics (Hons.) Examination, 2023

(2nd Year, 2nd Semester)

Mathematical Physics III

Paper – UG/Sc./Core /Phy/ Th/08

Time : 2 hours Full Marks : 40

Use **separate answer script** for each group

GROUP A

Answer any TWO $(10 \times 2 = 20)$

1. (a) If $i^{x+iy} = x + iy$ prove that that $x^2+y^2 = e^{-(4n+1)\pi y}$. (3)

   (b) Explain single valued and multivalued function of complex variable taking at least **one** example of each. (3)

   (c) Show that $e^x(x\cos y - y\sin y)$ is a harmonic function. Find the analytic function *f(z)* for which $e^x(x\cos y - y\sin y)$ is the imaginary part by finding out the conjugate function. (4)

   [CO 1]

2. (a) State the Laurent's series expansion of a function *f*(z) of complex variable about an isolated singularity z_0 and hence calculate the coefficient of negative power of z in that expansion. (4)

   (b) What is $Res[f(z), z_0]$? State its usefulness. (1+1)

   (c) Using Cauchy's Residue theorem evaluate the following integral

   $$\int_0^{2\pi} \frac{1}{1-2p\sin\theta+p^2}\,d\theta \quad (0 < p < 1)$$ (4)

   [CO 1]

3.(a) Expand $f(z) = \frac{z^2}{(Z+1)(Z+5)(Z+10)}$ about $z = -2$ in the region (i) $|z+2| < 1$. (4)

(b) Using contour integration and Jordan's Lemma evaluate $\int_{-\infty}^{\infty} \frac{\sin mx}{x}\,dx$. (6)

[CO 1]

[Turn over

(a). Consider the 3 following vectors in $\mathbb{R}^3$,: $v_1 = (1,1,1)$, $v_2 = (1,1,0)$ & $v_3 = (1,0,1)$. Check whether they are linearly independent. Use the Gram-Schmidt procedure to construct an orthornormal basis from these three vectors

(b) It is well known that the Carbon atom (C_4) has the shape of a regular *tetrahedron* embedded in $\mathbb{R}^3$. Using elementary vector analysis, establish that the $C-C$ valence bond angle is 109° 28" (approx.).

[4 + 6]

(a) Three radioactive nuclei (N_1, N_2, N_3) decay successively in series, so that the numbers $N_j(t)$ of the three types at any time obey the equations:

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

$$\frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3$$

If initially $N_1(0) = N_0$, $N_2(0) = 0$ & $N_3(0) = n$, find $N_3(t)$.

(b) Find the Laplace transform of the function: $f = e^{-\alpha t}$.

[8 + 2]

(a) Let $\mathbb{V}$ be the space of real polynomials (of a single variable x), *i.e.* $\mathbb{V} = \{p(x) : \deg p(x) \leq 2\}$. Define an inner product on $\mathbb{V}$ by:

$$< p, q > = \int_0^1 p(x)q(x)dx$$

Using the Gram-Schmidt process on the standard basis $(1, x, x^2)$ to obtain an orthnormal basis for $\mathbb{V}$.

(b). Given the 2×2 matrix $\mathbf{A}$, find the expression for the matrix $\mathbf{A}^4$, where the matrix $\mathbf{A}$ is given by:

$$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$$

[7 + 3]

(a). Define a Hermitian matrix. If A is a Hermitian matrix and C and D are Unitary matrices, then show that:

(*i*) $C^{-1}AC$ is Hermitian, (*ii*) $C^{-1}DC$ is Unitary.

(b). Suppose $\{\vec{X}_1, \vec{X}_2, \vec{X}_3, \ldots \vec{X}_k\}$ is a set of vectors in $\mathbb{R}^N$. Find the necessary condition for these vectors to be *linearly independent.*

(c). If $\mathbf{M}$ is the matrix given by:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

then find e^M.

[5 + 2 + 3]