Ex/SC/PHY/UG/CORE/TH/05/2023

B. Sc. Physics (Honours) Examination, 2023

(2nd Year, 1st Semester)

Mathematical Methods - II

Paper – CORE 05

Time : Two hours Full Marks : 40

(20 Marks for each group)

Use separate Answer Scripts for Groups A and B

Group - A (Mathematical Methods)

Answer any TWO questions.

1. Consider the following differential equation for the function $\psi(x)$:

$$\frac{d^2\psi(x)}{dx^2} + A\psi(x) = f(x)$$

where $\psi(x)$ vanishes in the limit $|x| \to 0$. Define the Green's function for the above equation and express the general solution $\psi(x)$ in terms of it. Hence explain how the method of Green's function works for solving inhomogeneous PDEs.
Marks: 10

2. Consider the wave equation in one dimension:

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Phi(x,t) = 0 \qquad (1)$$

By choosing a new set of variables $(x,t) \to (\zeta,\eta)$, show how the general solution of the wave equation can be written down (where $\zeta = x+ct$ and $\eta = x-ct$). Explain the physical principles and intuitions behind the assumption (case-wise / example-wise) of expecting solutions (of PDEs) in the form of *separated variables* to exist.
Marks: 10

3. Write down the three-dimensional Laplace's equation in spherical polar coordinates. assuming that a solution in the form of separated variables exists, obtain the equations that the radial, angular (polar) and the azimuthal coordinates satisfy. (You need not solve the individual equations explicitly). Hence write down the most general solution and discuss the interior and outer regions and hence discuss the continuation to the $r \to 0$ and $r \to \infty$ limits.
Marks: 10

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Group - B (Mathematical Methods)

*Answer any **TWO** questions.*

1. Use the generating function for Hermite polynomials to establish the recurrence relation connecting $H_{n+1}(x)$, $H_n(x)$ and $H_{n-1}(x)$. Use this relation along with the orthonormality relation of Hermite polynomials to find the value of the integral $\int_{-\infty}^{\infty} dx.[xH_n(x)]^2.e^{-x^2}$.
Marks: 4 + 6 = 10

2. (a) Find the radius of convergence of the equation $(1-x^2)y'' - 2xy' + p(p+1)y = 0$, where p is a positive number and the primes denote derivatives with respect to x, and show that $x = 0$ is an ordinary point of this differential equation.
(b) Use the Rodrigue's formula for Legendre polynomials to establish the orthonormality relation for these polynomials.
Marks: 5 + 5 = 10

3. Consider the differential equation $\ddot{x}(t) + 9x(t) = f(t)$, where the dots denote derivatives with respect to t.
(a) Which one among the following forms of $f(t)$ leads to resonance: (i) $f(t) = A\sin^2 t\cos t$ and (ii) $f(t) = A\sin t\cos t$?
(b) Evaluate the particular integral for the forcing function that produces resonance and identify the divergent term(s).
Marks: 3 + 7 = 10