Bachelor of Science Examination, 2023

First Year, First Semester

Subject: Mathematical Physics I

Physics (Paper- CORE 1)

Time: Two hours   Full Marks: 40

GROUP -A

Answer Question No. 1 and any two from the rest. ($1 \times 12 + 2 \times 4 = 20$).

1. Answer any Four ($4 \times 3 = 12$)

(i) If $u = 3x + 2y - z, v = x - 2y + z$ and $w = x(x + 2y - z)$ then show that u, v, and w are functionally related with each other. State the relevant property of Jacobian if used.

(ii) Define a homogeneous function of degree n in x and y. If z is a homogeneous function of degree n in x and y show that

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z \ .$$

(iii) Using Taylor's theorem find the conditions of maximum or minimum of a function $f(x, y)$.

(iv) Solve : $(1 + y^2)dx - (\tan^{-1} y - x)dy = 0$

(v) Reduce the Cauchy's homogeneous linear equation $\left(x^n \frac{d^n y}{d^n x} + k_1 x^{n-1} \frac{d^{n-1} y}{d^{n-1} x} + .. + k_n y = X\right)$. to a linear equation with constant coefficients.

[Turn over

2. Using Leibnitz's rule, evaluate $\int_0^{\alpha} \frac{\log(1+\alpha x)}{1+x^2} dx$.

3. Prove: $\frac{1}{f(D)}\left(e^{ax}V(x)\right) = e^{ax}\frac{1}{f(D+a)}V(x)$ and hence find out only the P.I. part of the solution of $(D^2 - 2D + 4)y = e^x \cos x$. (Symbols have their usual meaning).

4. Plot the following function and represent it by a Fourier Sine series

$$f(t) = t, \qquad 0 < t \leq \frac{\pi}{2}$$
$$= \frac{\pi}{2}, \qquad \frac{\pi}{2} < t \leq \pi$$

Group B

Answer **any four** questions from group B

1. A vector field A is defined as $\mathbf{A} = (-3 + 2xy)\,\mathbf{i} + x^2\,\mathbf{j} + z^2\,\mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are unit vectors along x, y, and z axes.

   (a) Find Divergence and Curl of **A**.

   (b) Two paths C_1 and C_2 are in the $x - y$ plane. Path C_1 connects point $(0, 0, 0)$ to $(1, 0, 0)$ and then to $(1, 1, 0)$ by two straight line segments. Whereas path C_2 connects $(0, 0, 0)$ to $(1, 1, 0)$ along a direct straight line. Find the line integrals of A from $(0, 0, 0)$ to $(1, 1, 0)$, along both the paths.

   (c) Argue if the results found in (a) and (b) above are consistent. (2+2+1)

2. (a) Define cross product between two vectors in terms of Levi-Civita tensor, which is defined as

$$\varepsilon_{ijk} = \begin{cases} +1 & \text{if } (i, j, k) \text{ is a cyclic permutation of } (1, 2, 3) \\ -1 & \text{if } (i, j, k) \text{ is a cyclic permutation of } (3, 2, 1) \\ 0 & \text{if any two of } i, j, k \text{ are equal} \end{cases}$$

   (b) Using the relation between repeated ε_{ijk} simplify $\vec{A} \times (\vec{B} \times \vec{C})$.

   (c) Using the result found in (b) above, obtain a simplification for Curl (Curl $(\vec{F})$) in terms of Divergence, Laplacian etc. (1+3+1)

[Turn over

3. (a) Find the eigenvalues and eigenvectors of the following matrix

$$\mathbf{A} = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 2 \\ 0 & 0 & 2 \end{pmatrix}$$

(b) Verify whether these eigenvectors are orthogonal to each other. (4+1)

4. A probability density function has the form

$$f(x) = Ae^{-\alpha(x-\mu)^2}, \quad \text{for } -\infty < x < +\infty.$$

where A and α are positive constants.

(a) Find an expression for the "normalization constant" A.

(b) Find the standard deviation of x. (2+3)

5. (a) The Cartesian coordinates x, y, z of a point P are expressed in terms of three curvilinear coordinates u_1, u_2, u_3, where $u_k = u_k(x, y, z)$, $k = 1, 2, 3$. Find the **unit vectors** along the u_1, u_2, u_3 curves, which represent the direction of increase of the respective generalized coordinates.

(b) Hence find an expression for the gradient in terms of these generalized curvilinear coordinates. (3+2)