3. a) Derive the Killing equation starting from the definition of a Kill vector.

   b) When a space is said to be maximally symmetric? How many distinct Killing vector fields are there for a maximally symmetric space?

   c) How you characterized a space geometrically if it is both homogeneous and isotropic? 4+2+2+2

4. a) Derive the condition for a vector to be hyper surface orthogonal. How the condition be modified when it is a Killing vector field? 4+2+2+2

   b) Distinguish the stationary and static space-time both in co-ordinate dependent and co-ordinate independent way. Does a stationary space-time implies to be a static one or vice-versa? Justify. (3+2)+(3+2)

5. Give a geometrical picture of Hamiltonian dynamics. What is a Hamiltonian vector field? Determine the Hamiltonian vector field corresponding to a function $f(q,p)$ in phase space. 3+3+4

6. Give a geometrical interpretation of the cosmological principle and Weyl postulate. Hence deduce the line element for FLRW model. Write down the Friedmann equations and the matter conservation equation for FLRW model. 2+5+3

M. Sc. Mathematics Examination, 2023

(2nd Year, 2nd Semester)

Mathematics

Paper – DSE-06 (B10)

[Application of Differantial Geometry to Relativity and Mechanics]

Time : 2 hours Full Marks : 40

The figures in the margin indicate full marks.

Notations / Symbols have their usual meanings.

Answer ***any four*** questions.

All questions carry equal marks.

1. Write down the general Lorentz transformation when the relative velocity between two inertial frames is along a unit vector $\vec{e}$. Obtain the velocity transformation law in vector form. Hence deduce the velocity addition formula. 2+4+4

2. a) What do you mean by “locally inertial” co-ordinate system? Does curvature tensor vanish identically in this co-ordinate system? Justify.

   b) Derive a first integral of the differential equation for geodesic. Hence introduce the notion of time-like/ null geodesics. Show that acceleration vector is zero along the geodesic. (2+1)+(3+1+3)