M. Sc. Mathematics Examination, 2023

(2nd Year, 2nd Semester)

Advanced Numerical Analysis (Theory)

Unit – 4.1

Time : One hour $\hspace{4cm}$ Full Marks : 20

Symbols & Notations have their usual meanings.

Answer any *Two* questions.

1. Use Ritz method to solve the two-point boundary value problem

   $$u'' + x^2 = 0 \text{ for } 0 < x < 1,$$

   subject to the boundary conditions $u(0) = 1$, $u'(1) + 2u(1) = 1$ with quadratic trial functions. $\hspace{1cm}$ 10

2. Describe Galerkin method for the boundary value problem: $Lu = f$ in $\mathcal{R}$ and hence find a one-parameter approximate analytical solution of the non-linear equation

   $$-2u\frac{d^2u}{dx^2} + \left(\frac{du}{dx}\right)^2 = 4 \text{ for } 0 < x < 1,$$

   subject to the boundary conditions $u(0) = 1$, $u(1) = 0$. $\hspace{3cm}$ 10

3. Solve the Volterra integral equation

   $$f(x) - \int_0^x (1 + 2x - 3u) f(u)\, du = x + \frac{2}{3}x^3$$

   For $x = 0\ (0.2)\ 0.4$ by approximating the integral using Trapezoidal rule. $\hspace{1cm}$ 10