M. Sc. Mathematics Examination, 2023

(2nd Year, 2nd Semester)

Mathematics

Paper – DSE-07 (B15)

[Theory of Semigroups - II]

Time : 2 hours Full Marks : 40

The figures in the margin indicate full marks.

Notations / Symbols have their usual meaning.

Answer any **four** questions.

1. a) Define O-simple semigroup. Prove that every finite O-simple semigroup is completely O-simple.

   b) Supposte S is a semigroup. Then show that S is O-simple if and only if for every a, b in S$\{0\}$, there exist x, y in S such that $xay = b$. 6+4

2. a) Suppose S is O-simple and contains a O-minimal left ideal, then show that S is the union of its O-minimal left ideals.

   b) Let ρ be a congruence on an inverse semigroup S : Then prove that S/ρ is a group if and only if $tr\rho = E \times E$; when E is set of all idempotent of S.

   5+5

3. Construct a Rees Matrix semigroup over a O-group. Prove that the semigroup is a completely O-simple ie is O-simple and has a primitive idempotent. 10

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4. Let S be a semigroup. Prove that the following are equivalent

   i) S is an inverse semigroup;

   ii) S is regular, and its idempotents commute.

   iii) every α-class and every R-class contains exactly one idempotent;

   iv) every element of S has a unique inverse. 10

5. Let S be an inverse semigroup with semilattice E of idempotents. Suppose ρ is a congruence on S. Prove that $(\ker\rho,\ tr\ \rho)$ is a congruence pair. Conversely, suppose (N,τ) is congruence pair. Prove that the relation $\rho(N,\tau) = \left\{(a,b) \in S \times S : \left(a^{-1}a,\ b^{-1}b\right) \in \tau, ab^{-1} \in N\right\}$ is a congruence on S. Moreover, prove that $\ker\ \rho(N,\tau) = N$, $tr\ \rho(N,\tau) = \tau$ and $\rho(\ker\ \rho,\ tr\ \rho) = \rho$. 10

6. a) Suppose $\phi : S \to T$ is a semigroup morphism from S onto T, where S onto T, where S be an inverse semigroup and T be a semigroup. Then prove that T is an inverse semigroup. Moreover, prove that ϕ is an inverse semigroup morphism.

   b) Let S be a O-simple semigroup containing at least one O-minimal left ideal and at least one O-minimal right ideal. Prove that for every O-minimal left ideal L, there exists a O-minimal right ideal R of S, s.t. (1) LR=S and (2) the identity element e of RL is a primitive idempotent.