M. Sc. Mathematics Examination, 2023

(2nd Year, 2nd Semester)

Mathematics

Paper – DSE-07 (B26)

[Stochastic Processes]

Time : 2 hours Full Marks : 40

The figures in the margin indicate full marks.

Notations / Symbols have their usual meanings.

Attempt ***any Five*** questions.

Each question carries 8 marks.

1. Consider a Markov Chain having state space $S=\{0,1,...,6\}$ and transition matrix P given below:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} \frac{1}{8} & 0 & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{7} & 0 & \frac{6}{7} \\ 0 & 0 & 0 & 0 & \frac{5}{6} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \end{bmatrix} \end{array}$$

a) Find all the recurrent classes and the set of transient states.

b) Find ρ_{0y}, $y = 0, 1, \ldots, 6$.

2. A gambler playing roulette makes a series of 10 Rupees bets. He has respective probabilities $\frac{9}{19}$ and $\frac{10}{19}$ of winning and losing each bet. He decides to quit playing as soon as his net winning reaches 250 Rupees or –100 Rupees.

a) Find the probability that the gambler comes out winning 250 Rs.

b) Find the expected winning amount of the gambler.

3. Suppose that every man in a village has exactly 3 children, which independently has probability 0·5 of being a boy or a girl. Suppose also that the number of males in the n-th generation forms a branching chain. Find the probability of extinction of the male line of a given man.

4. Let $\{X_n : n = 0, 1, 2, \ldots\}$ be a Markov chain with state space {0, 1, 2, ...} such that

$P[X_{n+1} = x+1 \mid X_n = x] = p \in (0,1)$ and

$P[X_{n+1} = 0 \mid X_n = x] = 1 - p$, for n = 0, 1, 2, ...

a) Prove that $\{X_n\}$ is irreducible.

b) Find $P_0(T_0 = n)$, $n \geq 1$.

c) Prove that $\{X_n\}$ is a recurrent chain and determine if it is positive recurrent or not.

5. Give an example (with proof) of

a) A positive recurrent Markov Chain.

b) A transient Markov Chain.

c) A Null Recurrent Markov Chain.

6. Consider a pure death process on S = {0, 1, 2, ...}

a) Find the Forward Equation.

b) Find $P_{xx}(t)$.

c) Find $P_{xy}(t)$ in terms of $P_{x,y+1}(t)$.

d) Find $P_{x(x-1)}(t)$.

e) Prove that if $\mu_x = \mu \cdot x$, $x \geq 0$ (μ constant), then

$$P_{xy}(t) = \begin{cases} {}^{x}c_y e^{-\mu t y} \left(1 - e^{-\mu t}\right)^{x-y} \\ 0 \text{ otherwise } \forall\ y = 0, 1, \ldots, x \end{cases}.$$