Ex/SC/MATH/PG/DSE/TH/06/B8/2023

# M. Sc. Mathematics Examination, 2023

( 2nd Year, 2nd Semester )

## Mathematics

## Paper – DSE-06

## [ Solid Mechanics III ]

Time : 2 hours Full Marks : 40

*The figures in the margin indicate full marks.*

*Notations / Symbols have their usual meaning.*

Answer any ***two*** questions from Group–A
and any ***two*** from Group–B

### Group – A

1. Deduce the differential equation for small bending of thin circular plate in the form

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)=\frac{p}{D}$$

where $p(r,\theta)$ is the intensity of the load, $w$ is the deflection in radial direction and $D$ denotes the flexural rigidity of the plate. 10

2. State and discuss Kirchoff's assumption attributed for the problem of bending of this plates. Describe the types of boundary conditions those may be considered for this problem. Also show that for the case of free edge the twisting moment and shearing force at the edge may be replaced by a single condition. 10

[ Turn over

3. a) Obtain solution of the two-dimensional biharmonic equation $\nabla_1^4 \omega = 0$ in the form of analytic functions.

b) Obtain the stresses and displacements in the absence of body force in the form

$$\sigma_x + \sigma_y = 2\left[F'(z) + \bar{F}'(\bar{z})\right]$$

$$\sigma_y - \sigma_x + 2i\,\tau_{xy} = 2\left[\bar{z}F''(z) + \chi''(z)\right]$$ 10

**Group – B**

4. Find the deflection $\omega$ of a circular plate of radius $a$ subjected to a uniformly distributed load of intensity $q$. The edge of the plate $r = a$ is clamped. Prove that maximum deflection $\omega_{\max} = \dfrac{qa^4}{64D}$ occurs at the centre of the plate. Also calculate bending moments $M_r$ and $M_\theta$ at $r = 0$ and $r = a$. 10

5. Starting from the stress equation of motion and the generalized heat conduction equation and also using the constitutive relations, deduce the fundamental energy equation in the form

$$\frac{d}{dt}(K + W + P) + \chi_T = \int_B x_1 v_1 dv + \int_A p_1 v_1 dA + \frac{c_v}{T_0}\int_B Q\,\theta dv + \frac{\lambda_0}{T_0}\int_A \theta\theta_{,n} dA$$



where

$K$ = kinetic energy,

$W$ = isothermal strain energy,

$P$ = heat energy function,

$\chi_T$ = dissipation function,

$x_1$ = internal body force component,

$v_1$ = velocity component,

$c_v$ = specific heat of the solid at constant volume,

$\lambda_0$ = thermal conductivity,

$T_0$ = constant reference temperature,

$p_1$ = $x_1$-component of surface traction,

$\theta$ = temperature increase,

$Q$ = heat source term 10

6. Solve one dimensional coupled thermoelastic half-space in the Laplace transform domain, when the boundary is subjected to

$$\rho v^2 u_x = H(t) \text{ and } T = 0 \text{ when } x = 0.$$

Where $\rho$ is the density, $v$ is the dilation wave speed, $H(t)$ is the Heaviside function, $u$ and $T$ denote respectively the displacement and temperature of the medium. 10