MASTER OF SCIENCE EXAMINATION - 2023

(2nd year, 1st Semester)

Mathematics

DSE 03 A8 (Solid Mechanics -I)

Full Marks: 40 **Time: Two Hours**

The figures in the margin indicate full marks.

(Symbols/Notations have their usual meanings)

Part- 1

Answer any two questions

1. Deduce generalized Hook's law in the form

$$\tau_{ij} = c_{ijkl}e_{kl,} \; i,j,k,l = 1,2,3.$$

Further show that due to symmetry of stress and strain tensors and existence of strain energy density function the number of coefficients in the generalized Hook's law reduces from 81 to 21. 14

2. A bar of length "l" and of rectangular cross-section is bent by two equal and opposite couples "M". Find the displacement components along the co-ordinate axes. Also find deflection curve of the axis of the beam. If the curvature is small, determine the shape of the edges of the vertical cross section of the bar after deformation. 14

3 a) Explain the physical interpretation of E, σ and μ. 5

b) Write Beltrami-Michell compatibility equations for stress components, hence show that when the components of the body force $\bar{F}$ are constants, then the stress and strain invariants θ and ϑ are harmonic functions and the stress components τ_{ij} and strain components e_{ij} are biharmonic functions. 5

c) Evaluate strain energy function U for the stress field (for an isotropic solid) $\tau_{11} = \tau_{22} = \tau_{33} = \tau_{12} = 0$ and $\tau_{13} = -\mu\alpha x_2, \tau_{23} = \mu\alpha x_1$, α is a constant and μ is the Lame's constant. 4

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Part - 2

Answer any one question

4. Prove that the components for plane problem in cylindrical polar co-ordinates can be expressed in the form

$$e_{rr} = \frac{\partial u_r}{\partial r}, \quad e_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}, \qquad e_{r\theta} = \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}.$$ 12

5. A disk of uniform thickness is rotating about an axis through the centre with angular velocity ω. Show that the stresses are greatest at the centre of the disk. Show also that by making a small circular hole at the centre of the disk, the maximum tangential stress approaches a value twice as great as that for a solid disk (assuming that there is no external forces applied at the boundaries). 12