3. a) Write down the Beverton-Holt fishery model and discuss the outcome of the result subject to different interpretations. 6

   b) What is 'Biological Over-Exploitation'? 2

M. Sc. Mathematics Examination, 2023

(2nd Year, 2nd Semester)

Mathematics

Paper – DSE-07 (B25)

[Renewable Bio-Economic and Impulsive Models on Biological Systems]

Time : 2 hours Full Marks : 40

The figures in the margin indicate full marks.

(Symbols and notations have their usual meanings)

(Use a separate Answer-Script for each Part)

Part – I (Marks: 24)

Answer ***any three*** from the following four questions.

1. a) Discuss an optimal control-induced mathematical model for the maximum production of biodiesel (BD) from *J. Curcus* oil considering the effect of mass transfer kinetics.

   b) Using the stirring effect as a control parameter $u(t)$, write down the suitable objective cost function and Hamiltonian for maximizing the BD production and minimizing the associated cost.

   c) How Pontryagin minimum Principles can be used in this control problem to evaluate the optimal stirring effect $u^*(t)$? Also, find out any three adjoint

equations corresponding to the controlled system.

1+2+(4+1)=8

2. a) Formulate a four dimensional mathematical model for pest management in *Jatropha Curcus* with integrated pesticides with suitable initial conditions.

   b) Find out the values of the coordinates of the interior equilibrium point and hence, describe the conditions for which a unique feasible equilibrium exists and two distinct feasible equilibria exist.

   c) Using Routh-Rurwitz criterion, discuss the local asymptotic stability of this formulated model.

2+3+3=8

3. a) Demonstrate a basic four-dimensional mathematical model of Psoriasis (without control) considering the interplay of the Th_1, Th_2, Th_{17} and Keratinocytes populations in detail.

   b) Discuss the positivity of the system solutions to show that the non-negative octant $\mathbb{R}^4_+$ is an invariant region.

   c) Also, check whether the solutions of the formulated model are bounded or not? If yes, provide the domain $\Omega \subseteq \mathbb{R}^4_+$ explicitly where the solutions are bounded. 2+3+3=8

4. a) Formulate a three-dimensional mathematical model for leprosy considering the infection of Schwann cells by M. leprae bacteria.

   b) Find the basic reproduction number $\mathcal{R}_0$ for this system using the well-known next generation matrix method. Find the disease-free equilibrium E_0 for the system and correlate the local asymptotic stability situation of E_0 with the $\mathcal{R}_0$ evaluated already.

   c) State briefly the essence of performing sensitivity analysis for the system using partial rank correlation coefficients (PRCC) method of the basic reproductive ratio. 1+(3+2)+2

Part – II (Marks: 16)

Answer ***any Two*** questions.

All questions carry equal marks.

1. a) What do you mean by 'Opportunity Cost' and 'Externality'? 3

   b) What is Production function? Discuss Cobb-Douglass production function. 5

2. a) What do you mean by 'Open Access Fishery'? 2

   b) What is 'Economic Rent'? 2

   c) Discuss the Gordon's model based on parabolic yield – effort curve and hence discuss the principle results. 4