7. Suppose that an item is ordered per week. The demand for the item during the week is uniform and with probabilities P(0) = 0.04, P(5) = 0.20, P(10) = 0.37, P(15) = 0.30, P(20) = 0.09. The carrying cost is Rs. 2/unit/week. The shortage cost is Rs. 24/unit/week. What should the optimal order level be at the beginning of each week? 8

8. When two probabilistic inventory systems are called equivalent? Show that the probabilistic order-level inventory systems with instantaneous and uniform demands are equivalent. 1+7

Ex/SC/MATH/PG/DSE/TH/06/B7/2023

M. Sc. Mathematics Examination, 2023

(2nd Year, 2nd Semester)

Mathematics

Paper – DSE-06 (B7)

[Production Planning and Inventory Control]

Time : 2 hours | Full Marks : 40

The figures in the margin indicate full marks.

(Notations/Symbolshave their usual meanings)

Answer ***any five*** questions.

1. Define economic production quantity (EPQ) for a production-inventory system. Derive the EPQ formula $Q^* = \sqrt{\dfrac{2c_3DP}{c_1(P-D)}}$, where the rate of replenishment is finite. Also derive the minimum cost formula

$$C_{\min} = \sqrt{2c_1c_3D\left(1-\frac{D}{P}\right)}$$ 1+5+2

2. Consider an order level system under the following assumptions:

(i) Demand rate (D) is deterministic and constant, (ii) Scheduling period (t_p) is a prescribed constant, (iii) The replenishment rate is infinite, (iv) At the beginning of each scheduling period, the replenishment size raises the inventory level to S, (v) Lead time is zero. Show that, for the system, the optimal order level $S^* = \dfrac{c_2 D t_p}{c_1 + c_2}$. If S is constrained to take the discrete values ... $-2u, -u, 0, u, 2u$, ... then S^* can be obtained by satisfying the relation $S^* - \dfrac{u}{2} \le \dfrac{c_2 D t_p}{c_1 + c_2} \le S^* + \dfrac{u}{2}$. 8

3. Suppose that an inventory system operates over a prescribed time horizon H and, during H, there exists a total demand for D units. Assume that there is no shortage in inventory. If n be the number of equal replenishments made during the period H then show that the optimal value of n i.e., n^* can be obtained by satisfying the relation

$$f\left(n^* - 1\right) \le \frac{c_1 DH}{c_3} \le f\left(n^*\right)$$

where $f(n) = \dfrac{n(n+1)}{(n+1)h(n) - nh(n+1)}$ and

$$h(n) = \frac{2}{3}n - \frac{\sqrt{1} + \sqrt{2} + \sqrt{3} + \cdots + \sqrt{(n-1)}}{n}$$

4. Consider a shop which produces and stocks three items. The items are produced in lots. The demand rate for each item is constant and can be assumed to be deterministic. No backorders are to be allowed. The pertinent data for the items are given in the following table:

Items	**1**	**2**	**3**
Demand rate (units/year)	10,000	12,000	7,500
Holding cost (Rs.)	20	20	20
Set up cost (Rs.)	50	40	60
Cost per unit (Rs.)	6	7	5

Determine approximately the economic order quantities when the total value of average inventory levels of three items is Rs. 1000. 8

5. Show that, for a stochastic inventory model with instantaneous demand and no set up cost, the optimum stock level S^* in discrete units can be obtained by satisfying the relation

$$\sum_{x=0}^{s-1} p(x) \le \frac{c_2}{c_1 + c_2} \le \sum_{x=0}^{s} p(x)$$ 8

6. Find the optimum order quantity for a product for which the price breaks are as follows:

Quantity	**Unit cost (Rs.)**
$1 \le q_1 < 500$	25.0
$500 \le q_2 < 1500$	24.8
$1500 \le q_3 < 3000$	24.6
$3000 \le q_4$	24.4

Given that the annual demand for a product is 500 units. The cost of storage per unit per year is 10% of the unit cost. The ordering cost is Rs. 180 for each order. 8