averages 4 minutes on each phase of the check up although the distribution of time spent on each phase is approximately exponential. If each patient goes through four phases in the check up and if the arrival of the patient to the doctor's clinic is approximately Poisson at an average rate of 3 per hour, then determine the average time and the most probable time spent in examination. 5+5

M. Sc. Mathematics Examination, 2023

(2nd Year, 1st Semester)

Mathematics

Paper – DSE-04-A26

[Probability, Games and Queuing Theory]

Time : Two hours Full Marks : 40

Part – I

Attempt ***any Two*** questions. Each question carries TEN (10) marks. 2×10=20

1. a) Define a field and a sigma-field with one example each case.

   b) Give an example of a field which is not a sigma field. 5+5=10

2. a) Prove that a characteristic function is uniformly continuous over the real line.

   b) State and prove Lindeberg-Levy's Central Limit Theorem giving clear mention of the results you are using. 2+8=10

3. a) Prove that if $\{A_n : n = 1, 2, ...\}$ is an infinite sequence of events in a probability space, then $\sum_{n=1}^{\infty} P(A_n)$ converges $\Rightarrow P\left[\limsup_{n\to\infty} A_n\right] = 0$.

b) Define a tail sigma field. Prove that if $\{A_n : n = 1, 2, ...\}$ is an independent sequence of events, then any tail event of this sequence is either a null event or a certain event. 5+5=10

Part – II (20 marks)

Answer ***any two*** questions.

1. a) Write down the minimax-maximin principle.

b) Let $f(x, y)$ be a real valued function of x and y such that $f(x, y)$ is a real number for $x \in A$ and $y \in B$; A, B being two sets. Suppose that both $\max_{x \in A} \min_{y \in B} f(x, y)$ and $\min_{y \in B} \max_{x \in A} f(x, y)$ exist, then the necessary and sufficient condition that

$$\max_{x \in A} \min_{y \in B} f(x, y) = \min_{y \in B} \max_{x \in A} f(x, y)$$

is that $f(x, y)$ possesses a saddle point. 2+8

2. a) For the game $\begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$, where a, b, c, d are all non-negative real numbers, prove that the optimal strategies are $A : \left(\frac{c+d}{a+b+c+d}, \frac{a+b}{a+b+c+d} \right)$, $B : \left(\frac{b+d}{a+b+c+d}, \frac{a+c}{a+b+c+d} \right)$

b) Players A and B play a game as follows: They simultaneously and independently write one of the three numbers 1, 2, 3. If the sum of the numbers written be even, then B pays to A this sum in rupees. If it be odd, then A pays the sum to B in rupees. Derive the matrix of the game for A and solve it. 5+5

3. a) Explain transient state and steady state in a queuing system.

b) Derive the differential-difference equations for the queuing model $(M / M / 1) : (\infty / FCFS)$. Obtain the steady state solution of the model and also find the expected value of the queue length. 2+8

4. a) A supermarket has two sales girls at the sales counters. The service time for each customer is exponential with a mean of 4 minutes, and the people arrive in a Poisson fashion at the rate of 10 an hour.

i) Calculate the probability that the customer has to wait for service.

ii) If a customer has to wait, what is the expected length of his waiting time?

b) A hospital clinic has a doctor who examines every patient brought in for a general check up. The doctor