M. Sc. Mathematics Examination, 2023

(2nd Year, 2nd Semester)

Mathematics

Paper – DSE-05A

[Numerical Analysis II]

Time : 1 hour 15 minutes Full Marks : 24

The figures in the margin indicate full marks.

Notations / Symbols have their usual meaning.

(Use separate answer script for each Part)

Part – I (Marks: 16)

Answer ***any two*** questions.

1. a) Determine the CFL condition for Lax-Wendroff formula.

   b) Using forward difference method compute the approximate solution of the PDE for three time steps:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2};\ 0 \le x \le 1,\ t > 0$$

subject to the initial and boundary conditions

$$u(x,0) = \frac{1}{2}\sin \pi x;\ u(0,t) = u(1,t) = 0$$

Use $h = \frac{1}{3}$ and mesh ratio parameter $\lambda = \frac{1}{2}$. 5+3

2. Use shooting method to solve the boundary value problem

$$\frac{d^2 y}{dx^2} = 2y\frac{dy}{dx},\ 0 < x < 1$$

$$y(0) = \frac{1}{3},\ y(1) = 0.5\,.$$

Assume $h = 0.25$ and two approximations to the slope as $s^{(0)} = 0.05$ and $s^{(1)} = 0.2$. Perform two iterations. 8

3. The heat conduction equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ is approximated by

$$\frac{1}{2k}\left(u_m^{n+1} - u_m^{n-1}\right) = \frac{1}{h^2}\left(u_{m-1}^{n} - 2u_m^{n} + u_{m+1}^{n}\right).$$

   i) Determine the truncation error.

   ii) Investigate the stability using Von Neumann method.

3+5

Part – II (Marks: 8)

Answer ***any one*** question.

1. The system of equations

$$x^2 + xy + y^2 = 7$$

$$x^3 + y^3 = 9$$

has a solution near $x = 1{\cdot}5$, $y = 0{\cdot}5$. Perform two iterations using Newton's method to obtain approximate root. 8

2. Solve the integral equation

$$f(x) - \frac{1}{2}\int_0^1 (x+u) f(u)\,du = x$$

numericall by approximately the integral using Trapezoidal rule after dividing the interval [0, 1] into two equal parts. 8