

**Master of Science Special Supplementary Examination - 2023**

**(Second Year, First Semester)**

**Mathematics**

**DSE - 02**

**Numerical Analysis - I (Theory)**

**Full Marks : 24**

**Time : One hour fifteen minutes**

Symbols / Notations have their usual meaning.

Use separate answer script for each part.

**Group - A**

**(16 Marks)**

Answer **any two** questions

1. (a) Determine the operational count of LU method for solving a linear system  $AX = b$ , where  $A$  is an  $n \times n$  coefficient matrix.

(b)

$$\text{Let } A(\alpha) = \begin{pmatrix} 0.1\alpha & 0.1\alpha \\ 1.0 & 1.5 \end{pmatrix}.$$

Determine  $\alpha$  such that the condition number  $\kappa(A)$  is minimized.

4 + 4

2. Prove that

$$(i) \quad \|X^{(k)} - X\| \leq \|H\|^k \|X^{(0)} - X\|,$$

$$\text{and } (ii) \quad \|X^{(k)} - X\| \leq \frac{\|H\|^k}{1 - \|H\|} \|X^{(1)} - X^{(0)}\|,$$

where  $H$  is an iteration matrix with  $\|H\| < 1$  and  $X^{(k)} = X^{(k-1)} + c$ ,  $k = 1, 2, 3, \dots$  with arbitrary choice  $X^{(0)} \in \mathbb{R}^n$ ,  $c \in \mathbb{R}^n$  and  $X = HX + c$ .

4 + 4

3. Find the Hermite polynomial of the third degree approximating the function  $y(x)$  such that  $y(x_0) = 1$ ,  $y(x_1) = 0$  and  $y'(x_0) = y'(x_1) = 0$ .

8

[ Turn Over

[ 2 ]

**Group – B**

**(8 Marks)**

Answer **any one** question

4. Using the Jacobi's method to find all the eigenvalues and the corresponding eigenvectors of the matrix

$$\begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}.$$

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5. (a) Prove that the largest eigenvalue in modulus of a square matrix can not exceed the largest sum of the moduli of the elements along any row or any column.

(b) Estimate the eigenvalues of the following matrix using Gerschgorin bounds:

$$\begin{pmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}.$$

5 + 3