Ex/SC/MATH/PG/DSE/TH/02A/2023

Master of Science Examination, 2023

(2nd Year, 1st Semester)

MATHEMATICS

UNIT - DSE 02

[Numerical Analysis - I (Theory)]

Time : 1.30 Hours Full Marks : 24

Symbols/Notations have their usual meaning.

Use separate answer script for each part.

GROUP - A

(16 marks)

Answer any ***two*** questions.

1. (a) Determine the operational count of Cholesky method for solving a linear system AX = b, where A is an n x n coefficient matrix.

   (b) Consider a system of equations

   $$x_1 - kx_2 = 2,$$
   $$-kx_1 + x_2 = 5,$$

   Where k is real constant. Then

   (i) for what values of k, the Jacobi and Gauss-Seidel iteration methods converge?

   (ii) for k = 0.25, determine the optimal relaxation parameter ω in SOR method. 4+2+2

[Turn over

2. (a) For any arbitrary $x^{(0)} \in \mathbb{R}^n$, prove that the sequence $\left\{X^{(k)}\right\}_{k=0}^{\infty}$ of approximations defined by

$$X^{(k)} = HX^{(k-1)} + C, \; \forall k \geq 1$$

converges to the unique solution of AX = b iff $\rho(H) < 1$.

(b) Suppose AX = b and $\tilde{A}\tilde{X} = \tilde{b}$ are two linear systems, where X(t) be the solution of the system A(t)X(t)=b(t). Then prove that

$$\frac{\|X-\tilde{X}\|}{\|X\|}\left(1-\kappa(A)\frac{\|A-\tilde{A}\|}{\|A\|}\right) \leq \kappa(A)\left(\frac{\|b-\tilde{b}\|}{\|b\|}+\frac{\|A-\tilde{A}\|}{\|A\|}\right)$$

where $\kappa(A) = \|A\|.\|A^{-1}\|$. 4+4

3. Define cubic spline interpolation and hence construct a natural cubic spline that passes through the points (1,3), (2,4) and (3,2). 3+5

GROUP - B
(8 marks)

4. (a) Let A = (a_{ij}) be a square matrix of order n whose eigenvalues are $\lambda_i (i = 1, 2, ..., n)$. If P_k be the sum of the moduli of the elements along the *k*th row excluding the diagonal element a_{kk}, then prove that

every eigenvalue of A lies inside or on the boundary of at least one of the circles $|\lambda_i - a_{kk}| = P_k$, k = 1, 2, ..., n.

(b) Estimate the eigenvalues of the following matrix using Gerschgorin bounds :

$$\begin{pmatrix} 10 & -1 & 0 & 1 \\ 0.2 & 8 & 0.2 & 0.2 \\ 1 & 1 & 2 & 1 \\ -1 & -1 & -1 & -11 \end{pmatrix}$$

5+3

5. For the matrix

$$\begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$$

(i) find all the eigenvalues and the corresponding eigenvectors.

(ii) verify that $S^{-1}AS$ is a diagonal matrix, where S is the matrix of eigenvectors. 4+4