Master of Science Examination, 2023
(2rd Year, 1st Semester)

MATHEMATICS

DSE-03 (A7)
(Nonlinear and Dynamic Programming)

Full Marks:40 Time: Two hours

The figures in the margin indicate full marks.
Symbols / Notations have their usual meanings

Use separate answer script for each Group

Group A (20 Marks)
Answer any two questions

S

1(a) Let $f : S \to R$ be a differential function on an open convex subset S of R^n. Then show that f is a convex function iff $f(x_1) - f(x_2) \geq (x_1 - x_2)^T \nabla f(x_2)$, $\forall\, x_1, x_2 \in S$.

(b) Use Lagrange multiplier method to solve
Minimize $f(x, y) = \frac{x^3}{3} - \frac{3y^2}{2} + 2x$
subject to $x - y = 0$.

5 + 5

2(a) Examine whether the following problem is a convex programming problem (CPP) or not:
Minimize $x_1 + x_2$
subject to $x_1^2 + x_2^2 \leq 1$ and $x_1^2 \leq x_2$.

(b) Let S be a non-empty open convex subset of R^n and $f : S \to R$ be twice differentiable on S. Then show that f is a convex function on S iff the Hesssian matrix $\nabla^2 f(x)$ is positive semi-definite $\forall x \in S$.

4 + 6

3(a) Solve the following optimization problem using Kuhn-Tucker (KT) conditions:
Maximize $z = 5 + 8x_1 + 12x_2 - 4x_1^2 - 4x_2^2 - 4x_3^2$
subject to $x_1 + x_2 \leq 1$
$2x_1 + 3x_2 \leq 6$.

(b) Minimize $f(x_1, x_2) = \frac{1}{3}(x_1 + 1)^3 + x_2$ subject to $x_1 \geq 1$, $x_2 \geq 0$,
using interior penalty function method with the calculus method of unconstrained minimization.

$6 + 4$

4(a) Explain quasiconvex and pseudoconvex functions with examples.

(b) Using Wolfe's method, solve the following quadratic programming problem (QPP):
Maximize $z = 2x_1 + x_2 - x_1^2$
subject to
$2x_1 + 3x_2 \leq 6$
$2x_1 + x_2 \leq 4$
$x_1, x_2 \geq 0$.

Group B (20 Marks)

Attempt any ***two*** questions. Each question carries 10 marks.

1. A thief enters a house to rob it. He can carry a maximum weight of 5 kgs into his bag. There are 4 items in the house with the following weights and values. What items should he take if he cannot divide any item into pieces? [Hint: Use DP approach to solve this 0/1 kmp sack problem] 10

Item	Weight (kg)	Value (rupees)
Mirror	2	300
Silver Nugget	3	400
Painting	4	500
Vase	5	600

2. Write a short note on Longest Common Subsquence problem with its uses and solve the following LCS problem using DP approach. 10

   S = ABAZDC

   T = BACBAD

3. Find a shortest path from (a) to (f) in the following network with respective distances (in kms). Is this shortest path unique? 10