MASTER OF SCIENCE EXAMINATION, 2023

(2nd Year, 1st Semester)

MATHEMATICS

UNIT - DSE 04 A25

[MODELLING OF BIOLOGICAL EVENTS]

Time : 2 Hours Full Marks : 40

Symbols/Notations have their usual meaning.

Answer any ***four*** questions.

1. The Lotka-Volterra predator-prey model with diffusion is represented by

$$\frac{\partial F}{\partial t} = \alpha F - \beta FG + D_1 \frac{\partial^2 F}{\partial x^2},$$

$$\frac{\partial G}{\partial t} = -\gamma G + \delta FG + D_2 \frac{\partial^2 G}{\partial x^2},$$

where F(x,t) and G(x,t) are the prey and predator densities at position x at time t. All parameters are positive and carry the usual meanings. Considering positive initial values and zero-flux boundary conditions, discuss the stability and diffusion-driven instability (if any) of the homogeneous coexisting equilibrium point. 10

2. Consider the following chemical reaction :

$$X + Y \underset{k_2}{\overset{k_1}{\rightleftharpoons}} P .$$

(a) Use the mass action law to construct the differential equations for the rate equations of the chemicals X, Y and P.

(b) What is the equilibrium constant of the reaction and what is its significance?

(c) If Y is replaced by X then describe the new rate equations. Prove in this case that the rate of production of P is half of the rate of change in X. What quantity is conserved in this reaction? 3+3+4

3. Suppose A and B be the densities of pery and predator populations at time t. If F(A) is the per capita growth rate of prey in the absence of its predator B, then construct a modified Lotka-Volterra type predator-prey model with suitable assumptions. If the predator has two life stages, viz. immature and mature, and only matured predators have the predation & reproduction capabilities, and τ is the maturation delay, then reconstruct your predator-prey model with the necessary assumptions. Find the stability criteria of the predator-free equilibrium of the structured model. 2+3+5

4. Write down a discrete mathematical mode of two competitive species for common resources. Determine the steady states and discuss the stability properties of the interior equilibrium. 1+2+7

5. Write down a discrete model of three species simple food chain model with their required assumptions. Discuss the steady states and discuss the stability properties of the equilibria. 1+2+7