M. Sc. Mathematics Examination, 2023

(2nd Year, 2nd Semester)

Graph Theory II (Theory)

Unit - 4.5 Paper – B 2.19

Time : $1\frac{1}{2}$ hours Full Marks : 30

Answer ***any five*** questions. 5×6

1. Define a *planar graph*. Let G be a connected planar graph with n vertices, e edges, f faces and k connected components. Show that $n-e+f-k=1$.

2. Define an *outerplanar graph*. Prove that the boundary of the outer face of a 2-connected outerplane graph is a spanning cycle. Hence show that the graph $K_{2,3}$ is planar but not outerplanar.

3. Define a *2-connected graph*. Prove that every minimal nonplanar graph is 2-connected.

4. Define a *tournament*. What is a *king* in a digraph? Prove that every tournament has a king.

5. Let $N=(V,E)$ be a single-source single-sink transport network with a flow F. Then show that F is a maximum flow if and only if there does not exist an F-unsaturated quasipath Q from the source vertex to the terminal vertex in N.

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6. What is a *matching* in a graph? Prove that a matching in a graph G is a maximum matching in G if and only if G has no M-augmenting path.

7. Define a *vertex cover* of a graph. Let $G=(V,E)$ be a graph. Show that $S\subseteq V$ is an independent set in G if and only if $V\setminus S$ is a vertex cover of G. Hence show that $\alpha+\beta=n$, where α is the maximum size of an independent set in G, β is the minimum size of a vertex cover of G and $n=|V|$.