

6. In the weighted graph with the following weight matrix, find the shortest path and its (weighted) length from the vertex a to the vertex z by Dijkstra's shortest path algorithm:

	a	v_1	v_2	v_3	v_4	v_5	v_6	z
a	0	3	4	∞	15	∞	∞	∞
v_1	3	0	∞	7	6	∞	∞	∞
v_2	4	∞	0	8	∞	4	∞	9
v_3	∞	7	8	0	∞	∞	4	5
v_4	15	6	∞	∞	0	3	7	∞
v_5	∞	∞	4	∞	3	0	∞	7
v_6	∞	∞	∞	4	7	∞	0	3
z	∞	∞	9	5	∞	7	3	0

M. SC. MATHEMATICS EXAMINATION, 2023

(2nd Year, 2nd Semester)

MATHEMATICS

PAPER – DSE-06 (B4)

[GRAPH THEORY - II]

Time : 1 hour 15 minutes

Full Marks : 24

All questions carry equal marks.

Answer any **four** questions.

4×6

1. Define a *plane graph* G and the *dual graph* G^* of G . Let G be a plane graph. Prove that G is bipartite if and only if G^* is Eulerian.
2. Define an outerplanar graph. Prove that every simple outerplanar graph has a vertex of degree at most 2.
3. State and prove Five Color Theorem for planar graphs.
4. Let $N = (V, E)$ be a single-source single-sink transport network with the source s , the sink t , a flow F and Q be an F -unsaturated quasipath from s to t in N with the slack $i(Q) = \lambda$. Let d be the value of the flow F . Then show that there exists a flow F^* in N whose value is $d + \lambda$.
5. Define a prufer sequence $\{2, 2, 3, 4, 1, 1\}$. Prove that the number of labeled trees of $n > 2$ vertices is n^{n-2} .

[Turn over