M. Sc. Mathematics Examination, 2023

(2nd Year, 1st Semester)

Mathematics

Paper – DSE-03 A5

[Fluid Mechanics-I]

Time : Two hours Full Marks : 40

The figures in the margin indicate the full marks.

Notations / Symbols have their usual meanings.

Answer question no. 1 and ***any two*** from the rest.

1. Show that if, the force field is conservative and there exists a functional relation between the pressure and density, the Euler's equation of motion for an inviscid fluid can be put in the form

$$\frac{\partial \vec{q}}{\partial t} - \left(\vec{q} \times curl\ \vec{q}\right) = grad\ \mathrm{H}$$

where $H = v + \int \frac{dp}{\rho} + \frac{q^2}{2}$.

Hence show that, if in addition the motion is irrotational, then the above form of the Euler's equation of motion is given by $-\frac{\partial \varphi}{\partial t} + v + \int \frac{dp}{\rho} + \frac{q^2}{2} = f(t)$. 8

or

Examine whether the motion specified by $\vec{q} = \frac{k^2\left(x\hat{j} - y\hat{i}\right)}{x^2 + y^2}$, (k = constant) is a possible motion for

an incompressible fluid. If so, determine the equation of the stream lines. Show that the motion is of potential kind and determine the velocity potential. 8

2. a) Define (i) Source, (ii) Sink and (iii) Doublet in two-dimensions. Show that the complex potential w for a single two-dimensional doublet of strength π at the origin and whose axis is directed along the x-axis is given by $w = \frac{\mu}{z}$. 7

   b) The particle velocity $\vec{q}$ for a fluid motion referred to a rectangular cartesian axes O(x, y, z) with unit vectors $\hat{i}, \hat{j}, \hat{k}$ is given by

$$\vec{q} = v_0\left(\hat{i}\cos\frac{\pi x}{2a}\cos\frac{\pi z}{2a} + \hat{k}\sin\frac{\pi x}{2a}\sin\frac{\pi z}{2a}\right)$$

   where v_0 is constant. Show that the pressure p associated with this velocity field in a liquid motion under no body force is given by

$$p = \frac{\rho v_0^2}{4}\left(\cos\frac{\pi z}{a} - \cos\frac{\pi x}{a}\right) + \text{constant}$$ 9

3. a) Show that the complex potential w, when an elliptic cylinder is rotating with angular velocity w in an infinite mass of liquid at rest at infinity, is given by $w = \frac{iw}{4}(a+b)^2 e^{-2\tau}$. 8

   b) A circular cylinder is fixed across a stream of velocity U with a circulation K round the cylinder. Show that the maximum velocity in the liquid is $2U + \frac{K}{2\pi a}$, where a is the radius of the cylinder. 8

4. a) Define vortex rows. Show that the motion due to a set of line vortices of strength K at points $z = \pm na$ (n=0,1,2,3...) is given by $w = \frac{iK}{2\pi}\log\left\{\sin\left(\frac{\pi z}{a}\right)\right\}$. 8

   b) Show that the velocity of propagation of surface wave of length λ in a canal of depth h is given by $c^2 = \left(\frac{g\lambda}{2\pi}\right)\tanh\left(\frac{2\pi h}{\lambda}\right)$ 8