M. Sc. Mathematics Examination, 2023

(2nd Year, 2nd Semester)

Mathematics

Paper – DSE-06 (B5)

[Fluid Mechanics II]

Time : 2 hours | Full Marks : 40

The figures in the margin indicate full marks.

Notations / Symbols have their usual meaning.

Answer question **no. 1** and any **three** from the rest.

1. Define Mach number. 1

2. a) State first law of thermodynamics. Show that the specific heat of a gas at constant volume is not equal to the specific heat of the gas at constant pressure.

   b) Show that while propagation of small disturbance in perfect gas, the disturbance in density propagates as a wave with a constant velocity *c*. (2+4)+7

3. Under what condition two geometrically similar flow pattern of perfect gas are said to be dynamically similar? Classify the steady and compressible flow at some location depending on Mach number. 10+3

4. Write down the basic equations of gas dynamics in integral form. Hence deduce the basic equation in differential form. Also show that $\frac{d}{dt}\left(\frac{p}{\rho^{\gamma}}\right)=0$. 2+8+3

5. a) State and prove Zemplen's Theorem. You may assume the relation

$$\frac{p_+}{p_-}=\frac{(\gamma+1)\rho_+-(\gamma-1)\rho_-}{(\gamma+1)\rho_--(\gamma-1)\rho_+}$$

   where p_+, ρ_+ and p_-, ρ_- are the pressure and density of the gas after and before crossing the discontinuity surface and γ is an adiabatic constant.

   b) Assuming Bernoulli's equation for steady two-dimensional gas motion in xy-plane in the form

$$\frac{1}{2}\vec{q}^{\,2}+\frac{\gamma}{\gamma-1}\Theta p^{\frac{\gamma-1}{\gamma}}=i_0(\psi),$$

   show that if Θ and i_0 are constants than the motion will be irrotational and conversely. (2+6)+5

6. Write down the equation for perturbed stream function for a supersonic gas flow past a thin profile. Show that the angle formed by the perturbation lines with the direction of unperturbed motion is given by $\alpha=\arcsin\frac{1}{M_\infty}$, M_∞ being Mach number for unperturbed stream. Show that the longitudinal and transverse components of perturbation velocity are porportional to the local angle of inclination of the streamlines of the perturbed motion to the direction of unperturbed flow. 1+2+10