M. Sc. Mathematics Examination, 2023

(2nd Year, 2nd Semester)

Mathematics

Paper – 4.5 (B-2.36)

[Epidemiology and Eco-epidemiology]

Time : 2 hours Full Marks : 50

The figures in the margin indicate full marks.

(Symbols and notations have their usual meanings)

(Use a separate Answer-Script for each Part)

Part – I (Marks: 30)

Answer ***any two*** questions.

1. a) Formulate a SIS epidemic model with vertical and horizontal transmission.

   b) Give a schematic diagram of your model.

   c) Find different equilibrium points of your model system and discuss the stability of nontrivial equilibrium point with basic reproduction number. 4+2+9

2. a) Describe a stochastic model of a general epidemic and formulate the differential difference equation for the model.

   b) Find the mean and variance in the number of infectives at any time. 8+7

3. a) With proper assumptions, formulate an epidemic model with latency.

   b) Give a schematic diagram of your model.

   c) Find different equilibrium points of your model system and discuss their stabilities. 4+2+9

Part – II (Marks: 20)

Answer ***any one*** question.

1. a) Formulate an eco-epidemiological model with disease present in prey population with Holling type I functional response with necessary assumptions.

   b) Write dimensionless form of your proposed model.

   c) Determine different equilibrium points of your model.

   d) Study the local stability of the interior equilibrium point. 7+4+4+5

2. Consider the following model:

$$\frac{dP}{dt} = rP\left(1-\frac{P}{K}\right) - \alpha P(S+I)$$

$$\frac{dS}{dt} = \alpha\alpha_1 PS - dS - \lambda SI,$$

$$\frac{dI}{dt} = \alpha\alpha_1 PI - (d+v)I + \lambda SI.$$ (P = Prey,

S = Susceptible Predator, I = Infected Predator)

   a) Give a flow diagram of the model.

   b) Determine different steady states of the model.

   c) Find the basic reproduction number around the disease free steady state of the model. Prescribe the conditions for stabilityof this point. Study the global dynamics of the model around disease-free steady state.

   d) Show that existence of positive steady state of the model implies its global stability. 2+4+8+6