M. Sc. Mathematics Examination, 2023

(2nd Year, 2nd Semester)

Mathematics

Paper – DSE-06 (B6)

[Epidemiology and Eco-epidemiology]

Time : 2 hours Full Marks : 40

The figures in the margin indicate full marks.

(Symbols and notations have their usual meanings)

(Use a separate Answer-Script for each Part)

Part – I (Marks: 16)

Answer ***any one*** question.

1. a) Formulate an eco-epidemiological model with disease in prey population stating necessary assumptions.

   b) Write dimensionless form of the model.

   c) Show that the existence of positive interior steady state of the model implies its local stability. 6+4+6

2. a) Formulate SVIR model with waning-vaccine induced immunity stating necessary assumptions.

   b) Give a flow diagram of your model.

   c) Find all feasible steady-states of the proposed model and discuss their qualitative behaviour.

   d) Find the critical waning rate to eradicate the disease of the model. 5+2+6+3

Part – II (Marks: 24)

Answer ***any three*** questions.

1. The basic SIR model with demography and vaccination expressed as

$$\frac{ds}{dt} = (\mu - \sigma)N - \beta IS - \mu S$$

$$\frac{dI}{dt} = \beta IS - \gamma I - \mu I$$

$$\frac{dR}{dt} = \gamma I - \mu R .$$

Find the underlying assumptions of this model. Draw the Schematic diagram and also find the stability conditions of the endemic equilibrium of the above model. 8

2. Write down a SIS model with the following assumptions:
   i) The horizontal transmission follows the law of mass action with K as the force of infection.
   ii) b, b' are the constant birth rate of susceptible and infected population respectively.
   iii) Progress of susceptible individuals are all susceptible at birth. Total offspring of infected individual, a fraction of p are susceptible and a fraction of q are infected at birth.
   iv) r and r' are death rate of susceptible and infected class respectively.
   v) The incubation time for infection and the gestation time are both zero.

Draw the schematic diagram of the model and find the stability conditions of the endemic equilibrium of the above model. 8

3. The population dynamics of measles is governed by

$$\frac{ds}{dt} = \mu N(1-P) - (\mu + \beta E)S$$

$$\frac{dE}{dt} = \beta ES - (\mu + \sigma)E$$

$$\frac{dI}{dt} = \sigma E - (\mu + \gamma)I .$$

Find the underlying assumptions of this model. Draw the schematic diagram and also find the stability conditions of the endemic equilibrium of the model. 8

4. Write down a SIRS model with the following assumptions:
   i) The horizontal transmission follows the law of mass action where β is the rate of infection.
   ii) The infected population recovers at a rate γ.
   iii) The recovered population loses the immunity and again becomes susceptibles at a rate δ.
   iv) No vertical transmission, no incubation periods are considered.
   v) No latent period.

Draw the schematic diagram of the model and find the stability conditions of the endemic equilibrium of the above model. 8