M. Sc. Mathematics Examination, 2023

(2nd Year, 1st Semester)

Mathematics

Paper – DSE04-A28

[Einstein's General Theory of Relativity]

Time : Two hours Full Marks : 40

Part – I

Answer ***any two*** questions. 4×10

All questions carry equal marks.

(Symbols/Notations have their usual meanings)

1. a) Derive Einstein field equations for gravity, using the logical arguments of Einstein (state the assumptions clearly)

   b) How many field equations are there in a ndimensional space-time? Are all the field equations independent? Justify. (4+2)+(2+2)

2. a) Write down the Schwarzschild line element in spherical polar coordinates. State the region for the validity of the above solution. What are the difficulties for describing the solution in the rest of the space-time?

   b) State Birkhoff's theorem. Interpret the parameter involved in the solution. (2+2+2)+(2+2)

3. a) What do you mean by (i) Maximal manifold (ii) geodesically complete manifold, give example of each type.

3. b) Show that a test particle takes infinite time (in coordinate time) to reach Schwarzschild event horizon while by finite proper time it reaches the singularity. (2+2)+(3+3)

4. Define blackhole from the point of view of space-time geometry. How many types of static blackholes are possible in Einstein gravity? Classify the space-time region for Reissner-Nordstram blackhole. 2+3+5

Part – II

Answer ***all*** questions. 20

1. Rewrite the Einstein line element for a static universe

$$ds^2 = dt^2 - \frac{dr^2}{\left(1-\frac{r^2}{R^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

in isotropic coordinate system...

or,

Show that the radius of the closest stable circular orbit for Schwarzschild spacetime is $6m$. 7

2. Show that in general space time, $J^+(p)$ is neither open nor a closed set.

or,

Show that for a spacetime (M, g_{ab}) to be stably causal is that a differentiable function f exists on M so that $\nabla^a f$ is a past-directed time-like vector field. 6

3. Show that the future Cauchy horizone $H^+(S)$ of an achronal set S is achronal and closed.

or,

Write Schwarzschild spacetime metric in Eddington - Finkelstein coordinates system. 7