M. Sc. Mathematics Examination, 2023

(2nd Year, 2nd Semester)

Mathematics

Paper – DSE-07

[Computational Fluid Dynamics - II]

Time : 1 hour 15 minutes Full Marks : 24

The figures in the margin indicate full marks.

Notations / Symbols have their usual meaning.

Answer **Q.No. 1** and any ***two*** from the rest. 4+2×10=24

1. Use Galerkin finite element method to find one parameter approximate analytical solution of the nonlinear equation

$$-2u\frac{d^2u}{dx^2}+\left(\frac{du}{dx}\right)^2=4 \text{ in } 0<x<1.$$

subject to the boundary conditions $u(0)=1$, $u(1)=0$. 4

2. Consider a domain $\Omega\subset\mathbb{R}^2$ with smooth boundary $\partial\Omega=\overline{\Gamma_u\cup\Gamma_q}$ and $\Gamma_u\cap\Gamma_q=\phi$. Derive the Galerkin formulation of the following boundary value problem:

$$\frac{\partial}{\partial x_1}\left(\frac{\partial u}{\partial x_1}\right)+\frac{\partial}{\partial x_2}\left(\frac{\partial u}{\partial x_2}\right)=f \text{ in } \Omega,$$

$$-k\frac{\partial u}{\partial n}=\overline{q} \text{ on } \Gamma_q,$$

$$u=\overline{u} \text{ on } \Gamma_u,$$

where $k=k(x_1,x_2)$ and $f=f(x_1,x_2)$ are continuous

functions defined in Ω as well as continuous functions $\overline{q}=\overline{q}(x_1,x_2)$ on Γ_q and $\overline{u}=\overline{u}(x_1,x_2)$ on Γ_u. 10

3. Define finite element and the number of degrees of freedom in FEM. Determine the shape functions for a typical triangular element by using linear Lagrange's polynomial and hence show that the shape functions are same as the area coordinates. 2+8

4. Use Rayleigh-Ritz method to find an approximate analytical solution of the Poisson's equation

$$-\nabla^2u=2(x+y)-4 \text{ in } \Omega=\left\{(x,y)\in\mathbb{R}^2:0<x,y<1\right\}$$

subject to the boundary conditions

$$u(0,y)=y^2,\ u(x,0)=x^2,\ u(1,y)=1-y,\ u(x,1)=1-x.$$

Use two parameters to set up the overall stiffness matrix. 10