M. Sc. Mathematics Examination, 2023

(1st Year, 1st Semester)

Mathematics

Paper – 1.4

[General Mechanics]

Time : 2 hours Full Marks : 50

The figure in the margin indicate full marks.

Symbols/Notation have their usual meanings.

Answer ***any five*** questions.

1. State the principle of least action. What do you mean by Legendre's dual transformation? Derive Hamilton-Jacobi partial differential equation. 2+3+5

2. a) Define generalized momentum corresponding to a generalized co-ordinate. What do you mean by a cyclic co-ordinate? Show that a cyclic co-ordinate is also absent in the Hamiltonian.

   b) The K.E. and P.E. of a particle are given by

   $T = \frac{1}{2}\left(x^2 + y^2\right)$ and

   $V = \frac{A}{x^2} + \frac{A'}{y^2} + \frac{B}{r} + \frac{B'}{r'} + C\left(x^2 + y^2\right)$,

   where A, A', B, B' and C are constants and r, r' are the distances of the particle from the points whose co-ordinates are $(C, 0)$ and $(-C, 0)$ respectively, C

being a constant. Show that the system can be derived as a Liouville's type. (1+2+2)+5

3. Write down Hamilton's equations of motion in symmetrical form. State the interelation between the Poisson bracket and the Lagrange's bracket. Prove the Jacobi identity for Poisson bracket. 2+2+6

4. a) Define canonical transformation. Show that the Jacobian of a canonical transformation is unity.

   b) Derive the infinitesimal change of the phase-space variables under infinitesimal canonical transformation.

   c) Examine whether the transformation $Q = \sqrt{q}\cos p$, $P = \sqrt{q}\sin p$ represents a canonical transformation or not. (2+2)+4+2

5. a) Starting from D'Alembert's principle, derive Hamilton's principle by using variational method.

   b) The K.E. and P.E. of a dynamical system with 2 degrees of freedom are given by

   $$T = \frac{q_1^2}{2(a+bq_2)} + \frac{1}{2}q_2^2\dot{q}_2^2 \ , \ V = c + dq_2$$

   where a, b, c and d are constants. Show that the value of q_2 in terms of time is given by the equation of the form $(q_2 - K)(q_2 + 2K)^2 = h(t - t_0)$ with h, k and t_0 as constants.

6. a) Derive the velocity and acceleration of a moving particle in 3D using spherical polar co-ordinates.

   b) Establish Euler's dynamical equation of motion. (3+3)+4

7. a) Show that the time period of small oscillation of a constraint physical system always lie between the corresponding time periods of the unconstrained system.

   b) Two uniform rods of same mass and same length $2a$ are freely jointed at a common extremity and rests on two smooth pegs which are in the same horizontal plane. Each rod is inclined at some angle α to the vertical. Show that the time of small oscillation when the join moves in a vertical straight line through the centre of the line joining the pegs is

   $$2\pi\sqrt{\frac{a}{9g}\left(\frac{1+3\cos^2\alpha}{\cos\alpha}\right)}$$ 4+6