M. Sc. Mathematics Examination, 2023

(1st Year, 1st Semester)

Mathematics

Paper – Core-04

[General Mechanics]

Time : Two hours Full Marks : 40

Answer any **four** questions.

All questions carry equal marks.

Notations and Symbols have their usual meanings.

1. a) If a co-ordinate is cyclic then will it be present in the Hamiltonian or not? Justify your answer. 3

   b) A particle moves in a plane under the action of two Newtonian centres of attraction at the points $(c, 0)$ and $(-c, 0)$, the attractions being $\frac{\mu}{r^2}$ and $\frac{\mu}{(r')^2}$ respectively; r, r' being the distances from $(c, 0)$ and $(-c, 0)$ respectively. Show that the problem is of Liouville's type. 7

2. a) Given $H = \frac{p^2}{2} - \frac{1}{2q^2}$, show that the quantity $G = \frac{pq}{2} - Ht$ is a constant of motion. 3

   b) Show that Poisson bracket is invariant under canonical transformation. 7

[Turn over

3. a) Obtain Hamilton's equations of motion for a system having n degrees of freedom from Hamilton's principle. 4

b) For a mechanical system described by n generalized coordinates $q_1, q_2 \ldots q_n$ show that the kinetic energy can be formulated as $T = T_2 + T_1 + T_0$, where $T_2 = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_{ij}\dot{q}_i\dot{q}_j$, $T_1 = \sum_{i=1}^{n}\beta_i\dot{q}_i$ and $T_0 = \gamma_0$. The quantities α_{ij}, β_i and γ_0 are to be determined by you. 4

4. a) Show that $J_1 = \int_{S_2}\sum dq_i dp_i$, is invariant under canonical transformation, where S_2 is a 2D surface in phase space.

b) Two heavy uniform rods AB and AC, each of mass m and length $2a$ are hinged at A and placed symmetrically over a smooth cylinder of radius c, whose axis is horizontal. If they are slightly and symmetrically displaced from the position of equilibrium, show that time of small oscillation is

$$2\pi\sqrt{\frac{a\cos\alpha}{3g}\left(\frac{1+3\cos^2\alpha}{1+2\cos^2\alpha}\right)}$$

where $a\sin^3\alpha = c\cos\alpha$. 6

5. a) State and prove conservation law of linear and angular momentum for a physical system. 3

b) Derive the Hamilton-Jacobi differential equation related to a dynamical system. 4

c) Examine whether the transformation :

$$Q = \log\left(1+\sqrt{q}\cos p\right),\ P = 2\left(1+\sqrt{q}\cos p\right)\sqrt{q}\sin p$$

is canonical or not. 3

6. a) Define Eukrian angles θ, ϕ and ψ.

b) Write down the Lagrangian of a symmetrical top in terms of these angles.

c) Using Lagrange's equation of motion, show that θ can be determined by solving an ordinary differential equation: $\ddot{\theta} + f(\theta) = 0$.

d) Discuss the stability of the symmetrical top when it executes steady motion at $\phi = \Omega$, a constant.

2+1+3+4