Ex/SC/MATH/PG/CORE/TH/05/2023

M. Sc. Mathematics Examination, 2023

(1st Year, 1st Semester)

Mathematics

Paper – Core-05

[Differential Geometry, Probability & Stochastic Process]

Time : Two hours Full Marks : 40

Part – I

(Differential Geometry)

Answer any **four** of the questions.

(Notations and Symbols have their usual meanings.)

1. i) Define unit vector in a Riemannian space. Check whether the vector with components $(1, 0, 0, v_2/c^2)$, c being constant is a unit vector or not in a space with line element given by

$$ds^2 = -\left(dx^1\right)^2 - \left(dx^2\right)^2 - \left(dx^3\right)^2 + c^4\left(dx^4\right)^2.$$

ii) If for $n > 2$, $R_{ij} - \frac{1}{2} g_{ij} R = 0$, show that $R_{ij} = 0$, whose R is scalar curvature and R_{ij} is Ricci tensor of an n-dimensional Riemanian space. 5

2. Find the curvature at any point of the curve $c : x' = a$, $x^2 = t$, $x^3 = bt$, where a, b are non-zero constants in a space with metric $ds^2 = \left(dx^1\right)^2 + \left(x^1\right)^2\left(dx^2\right)^2 + \left(dx^3\right)^2$.

[Turn over

3. Prove that necessary and sufficient condition for a given space curve to be a helix is that the ratio of the curvature to the torsion is constant. 5

4. Find the angle between two parametric curves on a surface and deduce the condition of the orthogonality of the parametric curves. 4+1

5. Find the differential equations of the geodesic on a surface with the metric $ds^2 = (du)^2 + (v^2 - u^2)(dv)^2$. 5

6. Define mean curvature of a surface. Find the mean curvature of the hyperbolic paraboloid $x^3 = x^1 x^2$. 1+4

Part – II

(Probability and Stochastic Processes)

Attempt any **two** questions.

Each question carries **Ten** marks.

1. A Gambler playing roulette makes a series of one hundred rupee bets. He has probability $\frac{5}{11}$ of winning each bet. The Gambler decides to quit gambling as soon as he wins 100 rupees or loses 100,000 rupees.

   a) Find the probability that at the time when the Gamble quits, he loses 100,000 rupees.

   b) Find the expected loss of the Gambler.

2. Let a fair coin be tossed N number of times. Let S(N) be the number of Heads obtained. Show that as $N \to \infty$, this

coin tossing experiment can be identified with the experiment of choosing a real number at random from the interval $(0,1]$. Further, show that, Given $\varepsilon > 0$,

$$\lim_{N\to\infty} P\left[\left|\frac{S(N)}{N} - \frac{1}{2}\right| \geq \varepsilon\right] = 0.$$

3. Let P be the one step transition matrix associated with a time-homogeneous Markov Chain with six states.

   a) Find the transient states of this chain.

   b) Find all the irreducible closed recurrent classes of this chain.

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{10} & 0 & \frac{9}{10} & 0 & 0 \\ \frac{9}{10} & 0 & 0 & 0 & \frac{1}{10} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & 0 \end{bmatrix}$$