b) Starting from the invariance of space-time interval, deduce the identity for velocities and hence show that mass varies in the Special Theory of Relativity. 4+(3+3)

4. Define Killing vector field. Find the Killing vectors for 3D Euclidean space. How many Killing vectors are there for FRW space-time? Define spherically symmetric space-time in terms of Killing vector fields. 2+4+2+2

5. Discuss Canonical transformation using differential forms. Show that, if $\vec{X}_f$ and $\vec{X}_g$ are Hamiltonian vector fields then $\vec{X}_{\{f,g\}}$ is also a Hamiltonian vector field. 5+5

6. Derive the condition for a vector to be hypersurface orthogonal. Give the definitions of stationary and static space-time both in co-ordinate dependent and in co-ordinate independent way. 4+3+3

7. Write down the space-time metric for homogeneous and isotropic model of the universe. Describe the geometrical structure of the space-time for three different choices of the curvature scalar. Write down the Einstein field equations and the conservation equation for this space-time model. 2+5+3

Ex/SC/MATH/PG/UNIT4.5/B 2.13/2023

M. Sc. Mathematics Examination, 2023

(1st Year, 1st Semester)

Mathematics

Unit – 4.5 (B-2.13)

[Differential Geometry and its Application - II]

Time : 2 hours Full Marks : 50

The figure in the margin indicate full marks.

Symbols/Notation have their usual meanings.

Answer ***any five*** questions. 5×10

1. a) Write down the differential equation for the geodesic in a Riemannian space with arbitrary parameter.

   b) Show that geodesic is a straight line both in E^2 and E^3.

   c) Show that acceleration vector is zero along the geodesic.

   d) If the metric coefficients are independent of any particular co-ordinate then prove that momentum along that direction is conserved. 2+3+2+3

2. Give an explicit derivation of the Newtonian limit of the Einstein equations and hence find the value of the coupling constant. 8+2

3. a) Show that the set of all Lorentz transformations along a direction forms a group. State clearly the binary operation involved?