4. Prove that the fractional knapsack problem has the greedy-choice property. 6

5. Write *Graham's scan* algorithm in pseudocode that solves the convex-hull problem by maintaining a stack of candidate points. 6

6. Write the recursive version of *depth-first-search* algorithm to traverse a graph. Find its worst-case time complexity. 6

7. Show the red-black trees that result after successively inserting the keys 41, 38, 31, 12, 19, 8 into an initially empty red-black tree. 6

M. Sc. Mathematics Examination, 2023

(1st Year, 2nd Semester)

Paper – DSE-01C

Design and Analysis of Algorithms

Time : $1\frac{1}{2}$ hours Full Marks : 30

(Symbols have usual meanings, if not mentioned otherwise)

Answer **Q.1** and ***any four*** from the rest.

1. Consider the problem of implementing a k-bit binary counter that counts upward from 0.

   a) Write a pseudocode for incrementing the counter.

   b) Use aggregate analysis to determine the amortized cost per operation. 2+4=6

2. a) Use induction to prove that radix sort works.

   b) Find the minimum and maximum numbers of elements in a heap of height h. 3+3=6

3. Consider a modification of the *rod-cutting problem* in which, in addition to a price p_i for each rod, each cut incurs a fixed cost of c. The revenue associated with a solution is now the sum of the prices of the pieces minus the costs of making the cuts. Give a dynamic-programming algorithm to solve this modified version of rod-cutting problem. 6