M. Sc. Mathematics Examination, 2023

(1st Year, 1st Semester)

Mathematics

Paper – Core-03

[Topology]

Time : Two hours Full Marks : 40

Answer any **five** questions.

Notations and Symbols have their usual meanings.

1. i) If A is a countable subset of $\mathbb{R}^2$ then show that $\mathbb{R}^2 \backslash A$ is connected.

   ii) Prove that a topological space X is disconnected iff there exists a continuous mapping from X onto the discrete two-point space $\{0, 1\}$. 4+4

2. i) State Ascoli-Arzela theorem. Use it to show that the set $S = \{x^n : n \in \mathbb{N}\}$ is not a compact subset of the space $C[0, 1]$.

   ii) Let $f : X \to Y$ be a bijective continuous function, where X is compact and Y is T_2. Then show that X and Y are homeomorphic. 4+4

3. i) Let X be a compact T_2 space such that every point of X is a limit point of X. Show that X is uncountable.

   ii) Consider the set of real numbers $\mathbb{R}$ with usual topology τ and lower limit topology τ_l. Show that τ_l is finer than τ. 4+4

[Turn over

4. i) Let A and B be two non-empty sets. If $f : A \to B$ and $g : B \to A$ be two injective functions then show that A and B have the same cardinal numbers.

   ii) Justify whether $\omega + 1 = 1 + \omega$ and $\omega.2 = 2.\omega$, where 1, 2, ω have their usual meaning as ordinal numbers. 4+4

5. i) Show that if a topological space is sequentially complete then it is countabily compact.

   ii) Let $S = \{x \in \mathbb{Q} : a < x < b\}$ and $T = \{x \in \mathbb{Q} : c \leq x \leq d\}$, where a, b are rational numbers and c, d are irrational numbers. Justify whether S and T are compact in $\mathbb{Q}$. 4+4

6. i) Let X, Y, Z be topological spaces. If $f : X \to Y$ and $g : Y \to Z$ are continuous then $g \circ f : X \to Z$ is continuous.

   ii) Show that every metric space is T_2. Give an example of a topological space which is not T_2. 4+4