where $E_{ij}\left(i, j = x, y, z\right)$

are Lagrangian finite strain tensor at (x, y, z).

b) What do you mean by infinitesimal strain? Deduce the expression for the infinitesimal strain components $e_{ij}\left(i, j = x, y, z\right)$. 7+3

4. a) Show that the volumetric strain is equal to the sum of the three longitudinal strains.

b) Give the geometrical interpretation of the longitudinal strain components e_{xx}, e_{yy}, e_{zz}. 5+5

5. a) Derive the equation of continuity in Lagrangian method.

b) What do you mean by constitutive equation? Write down the constitutive equations for (i) viscous fluid (ii) Elastic solid. 5+5

6. a) Stating the assumptions clearly, derive the Euler's equation of motion for a perfect fluid in the form

$$\frac{\partial \vec{q}}{\partial t} - \left(\vec{q} \times curl\ \vec{q}\right) = -\text{grad H}.$$

b) Examine whether the motion specified by

$\vec{q} = \dfrac{k^2\left(x\hat{j} - y\hat{i}\right)}{x^2 + y^2}$, k being a constant, is a possible motion for an incompressible fluid. Show that the flow is of potential kind and determine the velocity potential. 6+4

M. Sc. Mathematics Examination, 2023

(1st Year, 2nd Semester)

Paper – DSE-01B

Mechanics of Continua

Time : Two hours | Full Marks : 40

Answer ***any four*** questions.

1. a) Derive the Cauchy's formula involving the stress vector components and stress tensor components.

b) Given the following stress distribution :

$$\left(T_{ij}\right) = \begin{bmatrix} y & -z & 0 \\ -z & 0 & -y \\ 0 & -y & T \end{bmatrix}.$$

Find T such that the stress distribution is in equilibrium with the body force $\vec{F} = -g\hat{k}$. 6+4

2. a) Define :

i) Principal stress

ii) Principal direction

b) Show that the principal stresses are all real and the corresponding stress directions are mutually perpendicular. 2+8

3. a) Show that the fundamental measure of deformation of a body at a point (x, y, z) is given by

$$2\left(E_{xx}dx^2 + E_{yy}dy^2 + E_{zz}dz^2 + 2E_{xy}dxdy + 2E_{yz}dydz + 2E_{zx}dzdx\right)$$