B. Sc. Mathematics (Hons.) Examination, 2023

(3rd Year, 2nd Semester)

Mathematical Physics and Relativity

Paper – DSE-4C

Time : Two hours Full Marks : 40

The figures in the margin indicate full marks.

(*Symbols / Notations have their usual meanings.*)

Answer ***any four*** questions. 4×10=40

All questions carry equal marks.

1. a) Derive the general Lorentz transformation when the relative velocity is along any arbitrary direction. Hence deduce the Lorentz transformation when relative velocity is along the x-direction.

   b) In special theory of relativity derive the law of addition of velocity in the form : $U = u \oplus v$, clearly defining the binary composition $\oplus$.

2. a) Derive an expression for K.E. in STR. Deduce from it the expression for K.E. in Newtonian mechanics. Prove the relativistic energy momentum relation and hence show that $E = mc^2$.

   b) Examine whether the two points $\left(\frac{1}{c}, 0, 0, 0\right)$ and $\left(\frac{2}{c}, 2, 1, 1\right)$ in 4D space-time are causally connected or not.

3. a) Introduce the notion of light cone in STR both from geometrical and physical ideas. Define time-like vector and null vector in Minkowski geometry.

   b) A man moving along the *x*-axis of an inertial frames at a velocity *v*, observes a body of proper volume V_0 moving at a velocity *u* along the *x*-axis of frame S. Show that the volume of the body as measured by the man is $V_0\sqrt{\frac{(c^2-u^2)(c^2-v^2)}{(c^2-uv)}}$.

4. Define 4 velocity in Minkowskian geometry. Show that it is a time-like vector of constant magnitude. What will be the nature of the 4 acceleration vector? How the magnitude of the 4 acceleration vector is related to the geometry of the world line?

5. a) Show that for the set of all inertial frames moving relative to each other along a fixed direction, the Lorentz transformations set form an abelian group.

   b) A student draws a circle of radius *r* in the boad of a classroom. Another student moves along a direction parallel to a particular radius with constant velocity *u*. Discuss the shape of the circle to the 2nd student.

6. a) Determine the Jacobian of a typical Lorentz transformation.

   b) Show that $\square = \frac{1\partial^2}{c^2\partial f^2} - \nabla^2$ is invariant under Lorentz transformation.

   c) If *u* and *v* are two velocities in the same direction and V be their resultant velocity, given by $\tanh^{-1}\frac{V}{C} = \tanh^{-1}\frac{u}{c} + \tanh^{-1}\frac{v}{c}$, then deduce the law of composition of velocity.