B. Sc. Mathematics (Hons.) Examination, 2023

(3rd Year, 1st Semester)

Linear Programming & Game Theory

Paper – DSE-1A

Time : Two hours Full Marks : 40

Use separate Answer script for each Group.

Symbols / Notations have their usual meanings.

Goup – A (20 Marks)

Answer ***any two*** questions.

1. a) Find a basic feasible solution of the following system of equations

$$x_1 + 3x_2 + 2x_3 + 3x_4 = 10$$
$$2x_1 - x_2 + 4x_3 + 6x_4 = 16$$

b) Solve the following linear programming problem by Simplex method

Maximize $z = 5x_1 + 3x_2$

Subject to
$$x_1 + x_2 \le 2$$
$$5x_1 + 2x_2 \le 10$$
$$3x_1 + 8x_2 \le 12$$
$$x_1,\ x_2 \ge 0$$

5+5

2. a) Solve the following linear programming problem by two phase method

Maximize $z = 2x_1 - x_2 + 2x_3$

Subject to $x_1 + x_2 - 3x_3 \le 8$
$4x_1 - x_2 + x_3 \ge 2$
$2x_1 + 3x_2 - x_3 \ge 4$
$x_1,\ x_2,\ x_3 \ge 0$

b) Use Charne's M Method to solve the following linear programming problem.

Maximize $z = x_1 - 2x_1 + 3x_3$

Subject to $x_1 + 2x_2 + 3x_3 = 15$ 5+5
$2x_1 + x_2 + 5x_3 = 20$
$x_1,\ x_2,\ x_3 \ge 0$

3. a) Find the dual of the following linear programming problem.

Maximize $z = x_1 + 4x_2 + 3x_3$

Subject to

$2x_1 + 3x_2 - 5x_3 \le 2$
$3x_1 - x_2 + 6x_3 \ge 1$
$x_1 + x_2 + x_3 = 4$
$x_1,\ x_2 \ge 0,\ x_3$ is unrestricted in sign.

b) Use duality to solve the following linear programming problem

Maximize $z = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \ge 1$ 4+6
$x_1 + x_2 \le 7$
$x_1 + 2x_2 \le 10$
$x_2 \le 3$
$x_1,\ x_2 \ge 0$

b) Use dominance principle to reduce the following game to 2×2 game and hence solve it.

$$\begin{pmatrix} 1 & -1 & 0 \\ -6 & 3 & -2 \\ 8 & -5 & 2 \end{pmatrix}$$ 5+5

4. a) Show that every two-person zero sum game can be formulated as a linear programming problem.

b) Solve graphically the game whose payoff matrix is given below :

		B			
		B_1	B_2	B_3	B_4
A	A_1	2	2	3	–1
	A_2	4	3	2	6

5+5

the same if a constant is added to or subtracted from any row or column of the cost matrix.

b) A machine operator processes four types of items on his machine and must choose a sequence for them. The set up cost per change depends on the item presently on the machine and the set up to be made according to the following table:

To item

From item	A	B	C	D
A	∞	4	7	3
B	4	∞	6	3
C	7	6	∞	7
D	3	3	7	∞

If he processes each type of item once and only once each week, how should he sequence the items on his machine in order to minimize the total set up cost?

4+6

3. a) State maximin-minimax principle.

Let $f(x, y)$ be a real valued function of x and y such that $f(x, y)$ is a real number for $x \in A$ and $y \in B$; A, B being two sets. If both $\max_{x\in A} \min_{y\in B} f(x, y)$ and $\min_{y\in B} \max_{x\in A} f(x, y)$ exist, then show that $\max_{x\in A} \min_{y\in B} f(x, y) = \min_{y\in B} \max_{x\in A} f(x, y)$.

Goup – B (20 Marks)

Answer ***any two*** questions.

1. a) Show that the number of basic variables in a transportation problem is at most $(m+n-1)$, m and n being the number of origins and destinations. Also show that there always exists a feasible solution of a transportation problem.

b) A computer centre has got three programmers. The centre needs three application programmes to be developed. The head of the computer centre, after studying carefully the programmes to be developed, estimates the computer time in minutes required by the experts to the application programmes as follows:

Programmers

Programmes	A	B	C
1	120	100	80
2	70	90	110
3	110	140	120

Assign the programmers to the programmes in such a way that the total computer time is least. 6+4

2. a) Explain how an assignment problem can be treated as a linear programming problem. Show that the optimal solution to the assignment problem remains