that $\left|\frac{m}{n} - a\right| < \varepsilon$. [You have use Pigeon hole principle in your proof and you have to state it.] 5

4. a) Determine (using definition) whether the following POSETS are lattices or not :

   i) {S, I} where S = {1, 3, 6, 9, 12} and $\forall a, b \in S$, a I b if a divides b.

   ii) {S, I} where S = {1, 5, 25, 125} and $\forall a, b \in S$, a I b if a divides b.

   iii) {P(S), I} where P(S) is the power set of a given non-empty set S and $\forall a, b \in P(S)$, a I b if a is a subset of b.

   b) Give an example of a POSET which :

   i) has a min element but no max element.

   ii) has no min element and no max element.

3+2=5

B. Sc. Mathematics (Hons.) Examination, 2023

(3rd Year, 2nd Semester)

Graph Theory & Combinatorics

Paper – DSE-4A

Time : Two hours | Full Marks : 40

(Symbols have usual meanings, if not mentioned otherwise)

Part – I (25 Marks)

Attempt **Question 1** and any ***two*** from the rest.

1. a) What do mean by *boolean expression* and *boolean function*?

   b) Five soldiers, A, B, C, D, and E, volunteer to perform an important military task if the following conditions are satisfied.

   i) Either A or B or both must go.

   ii) Either C or E, but not both, must go.

   iii) Either both A and C go or neither goes.

   iv) If D goes then E must also go.

   v) If B goes then A and D must also go.

   Define variables A, B, C, D, E such that an unprimed variable will mean that the corresponding soldier has been selected to go. Determine the boolean expression that specifies the combinations of volunteers that can get the assignment.

c) Is the function

$$T(w,x,y,z)=\sum(0,1,2,3,4,6,7,8,9,11,15)$$

boolean? If the answer is negative then give your explanation supporting the answer, otherwise

i) find all prime implicants and indicate which are essential; and

ii) find a minimal expression for T and determine whether it is unique. 2+2+5=9

2. a) If a graph G has p points and $\delta(G)\geq(p-1)/2$, then prove that G is connected.

b) Prove or disprove: If G_1 and G_2 are bipartite, then so is $G_1 \times G_2$. 4+4=8

3. a) Prove that a tree with $p \geq 3$ points has a diameter 2 if and only if it is a star.

b) Define eulerian graph. If the set of lines of a connected graph G can be partitioned into cycles, then prove that G is eulerian. 4+4=8

4. a) Prove that any 3-connected planar graph is uniquely embeddable on the plane.

b) Prove that every 5-connected planar graph has at least 12 points. Construct one example of such a plane graph. 4+4=8

Part – II (15 Marks)

Attempt ***any three*** questions.

Each question carries 5 marks.

1. A fair coin is tossed repeatedly and independently until two back to back Heads are observed. Let X be the (random) number of tosses needed to get this first 'Double Heads'.

a) Find the mass function of X.

b) Find the mean of X. 3+2=5

2. Let N couples be present in a Saturday Dance Party. On the dance floor, people are allowed to dance only in Male-Female pairs.

a) How many possible N pairs can be formed with these N males and N females present on the day?

b) In a particular choice of such N-pairs, if the K-th male happens to be dancing with the K-th female (they originally formed the K-th couple entering the party, K = 1, 2, ..., N), we say there is a match at the K-th position. Among all possible choice of N-pairs as in (a), in how many choices, there will be exactly L matches? L = 0, 1, 2, ..., N–1, N. 1+4=5

3. Let $\varepsilon > 0$ and $a > 0$ be two given real numbers among which a is irrational.

Prove that there exist two natural numbers m and n such