4. a) What is a developable surface? Check whether the surface $x = f_1(u)$, $x^2 = f_2(u)$, $x^3 = v$ is developable or not, where f_1, f_2 are differentiable functions.

   b) The components of a contravariant vector in the (x^i) coordinate system are 8 and 4. Find its components in $(\overline{x}^i)$ coordinate system if $\overline{x}^1 = 3x^1$ and $\overline{x}^2 = 5x^1 + 3x^2$.

5. a) Assume that $A(p,q)B_{qj} = C_{pj}$ holds, where B_{qj} is an arbitrary tensor and C_{pj} is a covariant tensor of type (0, 2). Check whether $A(p,q)$ is a tensor or not. If so, what is the type of it?

   b) Find the Gaussian curvature of the surface $x = a\sin u \cos v$, $y = a \sin u \sin v$, $z = a\cos u$, where a is a constant and identify the surface. 4+6

6. a) Find the metric tensor and conjugate metric tensor for two-dimensional polar coordinates (r,θ).

   b) Prove that for Bertrand mates, $\tau\overline{\tau} =$ constant, where τ and $\overline{\tau}$ are torsion of the Bertrand mates. 4+6

B. Sc. Mathematics (Hons.) Examination, 2023

(3rd Year, 2nd Semester)

Differential Geometry

Paper – DSE-3C

Time : Two hours — Full Marks : 40

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

Answer ***any four*** of the following questions.

1. Find the equation of the involutes of a space curve $\vec{r} = \vec{f}(s)$. Also find the curvature of the involutes at some point. 4+6

2. Establish Serret-Frenet formulae for space curve. Hence find the relation between curvature and torsion of a space curve $\vec{r}(t) = (3\cos t,\ 3\sin t,\ 4t)$. 6+4

3. a) Identify the surface whose equation is given by $x = a\sin u\cos v$, $y = b\sin u \sin v$, $z = c\cos u$, where a, b, c are constants. Find the first fundamental form and unit surface normal of this surface.

   b) Find the angle between the parametric curves of this surface and expression of surface area between $u = u_1$ to $u = u_2$ and $v = v_1$ to $v = v_2$. 6+4