Ex/SC/MATH/UG/DSE/TH/04/B/2023(S)

B. Sc. Mathematics (Hons.) Examination, 2023

(3rd Year, 2nd Semester, Special Supplementary)

Bio Mathematics

Paper – DSE-4B

Time : Two hours Full Marks : 40

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

Part – I (Marks: 24)

Answer ***any three*** questions.

1. Deduce classical Lotka-Volterra Model of Prey-Predator Interaction stating its underlying assumptions. Show that all solutions of the model are periodic and structurally unstable. Write down also the basic drawbacks of the classical Lotka-Volterra model, if any.

2. Write down the single species time-delay model. Investigate the qualitative behaviour of the model and give your comments on the results.

3. Deduce the Gauss model of competition for two species sharing a common resources. Discuss the conditions for co-existence and extinction of either of the competing species.

4. Kolmogorov type Prey-Predator model is taken in the form :

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$$\frac{dx}{dt} = xg(x) - yp(x)$$
$$\frac{dy}{dt} = yq(x)$$

where the symbols have their usual meanings. Determine the steady states of this model and discuss their qualitative behaviour.

Part – II (Marks: 16)

Answer ***any two*** questions.

5. a) State Liapunov's stability theorem and Liapunov's function.

   b) Considering the linear system $\dot{x} = Ax$, $x \in \mathbb{R}^2$ with $\tau = \text{tracA}$, $\delta = \det A$, construct a bifurcation diagram $\tau - \delta$ plane by stating the conditions for different types of nature. 3+5

6. a) Use Cobwebbing method to sketch the solutions of the difference equation

$$x_{n+1} = \frac{\gamma x_n}{A + x_n} \cdot (\gamma,\ A > 0)$$

   with different initial values.

   b) If E is a open subset of $\mathbb{R}^2$ containing $(\bar{x}, \bar{y})$ and $f, g \in C'(E)$.

   State necessary and sufficient conditions for which the fixed point $(\bar{x}, \bar{y})$ of the nonlinear system

$$x_{n+1} = f(x_n, y_n)$$
$$y_{n+1} = g(x_n, y_n)$$

   is locally asymptotically stable. 6+2

7. a) Determine nature of the fixed point $\bar{x}$ of the system $x_{n+1} = f(x_n)$ when $|f'(x_n)| < 1$. Justify your answer.

   b) Consider a system

$$x_{n+1} = (r+1)x_n - rx_n^2 - bx_n y_n$$
$$y_{n+1} = Cx_n y_n + (1-d)y_n \quad (r, b, c \in \mathbb{R}^2,\ 0 < d < 1)$$

   Find all fixed points of this system.

   Determine the nature of the fixed points. 3+5