

B. SC. MATHEMATICS (HONS.) EXAMINATION, 2023

(3rd Year, 1st Semester, Supplementary)

PROBABILITY & STATISTICS

Paper - DSE-2A

Time : Two Hours

Full Marks : 40

Symbols / notations have their usual meanings.

Answer any eight questions.

$5 \times 8 = 40$

1. State and prove Tchebycheff's inequality for the case of discrete random variable having finite variance.
2. Define Empirical distribution. Show that the empirical distribution is the statistical image of the distribution of the population.
3. Two points P and Q are chosen at random on a line segment of length a . What is the probability that the distance between P and Q is less than b ($b < a$)?
4. Show that k-th order sample moment about the origin is a consistent and unbiased estimate of the k-th order moment about the origin of the population.
5. State and prove law of large numbers for the case of equal components.
6. Let X be normally distributed with mean m and standard deviation σ . Show that the sampling distribution of the sample mean is also normally distributed with mean m and standard deviation σ/\sqrt{n} for a sample of size n .
7. If the random variable X and Y are connected by the relation $aX + bY + c = 0$ then show that $\rho(X, Y) = -1$ if $ab > 0$.

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8. Find the maximum likelihood estimate of p for a Binomial (N, p) population.
9. Prove that every odd order central moment is zero for a symmetrical continuous distribution.
10. If $W = \{x : x \geq 1\}$ is the critical region for testing the hypothesis $H_0 : \theta = 2$ against the alternative hypothesis $H_1 : \theta = 1$ on the basis of the single observation x from the population having probability density function

$$f(x) = \theta e^{-\theta x}, \quad 0 \leq x < \infty$$

for the unknown parameter θ , then find the probability of Type-I error and the power of the test.

11. If X is a standard normal variate then prove that $\frac{X^2}{2}$ is $\Gamma\left(\frac{1}{2}\right)$ variate.